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ABSTRACT

A new basic algorithm is discussed that may be used to do factor analysis by any of these three methods: (1) unweighted least squares, (2) generalized least squares, or (3) maximum likelihood. (CK)

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# RESEARCH MEMORANDUM

RM-71-5

NEW RAPID ALGORITHMS FOR FACTOR ANALYSIS BY UNWEIGHTED LEAST SQUARES, GENERALIZED LEAST SQUARES AND MAXIMUM LIKELIHOOD

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Educational Testing Service
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May 1971

# NEW RAPID ALGORITHMS FOR FACTOR ANALYSIS BY UNWEIGHTED LEAST SQUARES, GENERALIZED LEAST SQUARES AND MAXIMUM LIKELIHOOD

Karl G. Jöreskog and Marielle van Thillo

## 1. Introduction

We shall describe a new basic algorithm that may be used to do factor analysis by any of the three methods

(i) unweighted least squares (ULS)

(ii) generalized least squares (GLS)

(iii) maximum likelihood (ML).

The ULS method produces solutions that are equivalent to those obtained by the iterated principal factor method and the minres method (see Harman, 1967, Chapters 8 and 9). The generalized least squares method is described by Jöreskog and Goldberger (1971). In the ML case, the new algorithm is simpler and faster than Jöreskog's (1967a,b) method UMLFA. It is similar to Clarke's (1970) algorithm but Heywood cases are handled in a simpler and more efficient way. Although the new algorithm handles ULS and GLS as well as ML, the computer program is shorter than UMLFA.

The GLS and ML methods are scale free. When multivariate normality is assumed, both GLS and ML yield estimates that are asymptotically efficient. Both GLS and ML require a positive definite variance-covariance matrix $S$ or correlation matrix $R$ ; ULS will work even on a matrix that is non-Gramian.

The model is the usual factor analysis model, which requires the population variance-covariance matrix or correlation matrix $\Sigma$ of the observed variables to be of the form

$$\Sigma = \Lambda\Lambda' + \psi^2 \quad , \tag{1}$$

where $\Lambda$ is a $p \times \kappa$ matrix of factor loadings and $\psi^2$ is a $p \times p$ diagonal matrix of unique variances. The factors are assumed to be orthogonal.

The model (1) is fitted to the observed variance-covariance matrix $S$ or to the corresponding correlation matrix $R$, by the minimization of a fitting function $F(\Lambda,\psi)$, which is different for each of the three methods. The minimization of $F(\Lambda,\psi)$ is done in two steps. First the conditional minimum of $F$ for given $\psi$ is found. This gives a function $f(\psi)$ which is then minimized numerically using the Newton-Raphson procedure. Function values and derivatives of $f$ of first and second order are given in terms of the characteristic roots and vectors of a certain matrix $A$. In the GLS and ML cases a transformation from $\psi_i$ to $\theta_i$ is made to obtain stable derivatives at $\psi_i = 0$. The basic formulas for ULS, GLS and ML are given in sections 2, 3 and 4 respectively.

## 2. Formulas for ULS

Fitting function: $F(\Lambda,\psi) = (1/2)\ \mathrm{tr}[(S - \Sigma)^2]$ , (2)

$$\Sigma = \Lambda\Lambda' + \psi^2 \ ,$$

$\Lambda'\Lambda$ is assumed to be diagonal (3)

Matrix whose roots and vectors are computed: $A = S - \psi^2$ (4)

Characteristic roots: $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_p$

Corresponding orthonormal vectors: $\omega_1,\omega_2,\cdots,\omega_p$

Conditional solution for $\Lambda$ for given $\psi$ : $\tilde{\Lambda} = \Omega_1\Gamma_1^{1/2}$ , (5)

$$\Gamma_1 = \mathrm{diag}(\gamma_1,\gamma_2,\cdots,\gamma_k) \ ,$$

$$\Omega_1 = [\omega_1\omega_2\cdots\omega_k]$$

Function minimized by the Newton-Raphson method:

$$f(\psi) = (1/2) \sum_{m=k+1}^{p} \gamma_m^2 \qquad (6)$$

First order derivatives: $\partial f/\partial\psi_i = -2\psi_i \sum_{m=k+1}^{p} \gamma_m\omega_{im}^2$ (7)

Second order derivatives:

$$\partial^2 f/\partial\psi_i\partial\psi_j = 4[\psi_i\psi_j \sum_{m=k+1}^{p} \omega_{im}\omega_{jm} \sum_{n=1}^{k} \frac{\gamma_m + \gamma_n}{\gamma_m - \gamma_n}\omega_{in}\omega_{jn} + \delta_{ij} \sum_{m=k+1}^{p} (\psi_i^2 - \gamma_m/2)\omega_{im}^2] \qquad (8)$$

Approximate second order derivatives: $\partial^2 f/\partial\psi_i\partial\psi_j \doteq 4\psi_i\psi_j(\sum_{m=k+1}^{p} \omega_{im}\omega_{jm})^2$ (9)

## 3. Formulas for GLS

Fitting function: $$F(\Lambda,\psi) = (1/2)\ \mathrm{tr}[(S^{-1}\Sigma - I)^2] \quad , \tag{10}$$

$$\Sigma = \Lambda\Lambda' + \psi^2 \quad ,$$

$\Lambda'\psi^{-2}\Lambda$ is assumed to be diagonal (11)

Matrix whose roots and vectors are computed: $A = \psi S^{-1}\psi$ (12)

Characteristic roots: $\gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_p$

Corresponding orthonormal vectors: $\omega_1,\omega_2,\cdots,\omega_p$

Conditional solution for $\Lambda$ for given $\psi$ : $$\tilde{\Lambda} = \psi\Omega_1(\Gamma_1^{-1} - I)^{1/2} \quad , \tag{13}$$

$$\Gamma_1 = \mathrm{diag}(\gamma_1,\gamma_2,\cdots,\gamma_k) \quad ,$$

$$\Omega_1 = [\omega_1\omega_2\cdots\omega_k]$$

Transformation: $$\psi_i = +\sqrt{e^{\theta_i}} \quad ; \quad \theta_i = \log \psi_i^2 \tag{14}$$

Function minimized by the Newton-Raphson method:

$$f(\Theta) = (1/2) \sum_{m=k+1}^{p} (\gamma_m - 1)^2 \tag{15}$$

First order derivatives: $$\partial f/\partial\theta_i = \sum_{m=k+1}^{p} (\gamma_m^2 - \gamma_m)\omega_{im}^2 \tag{16}$$

Second order derivatives:

$$\partial^2 f/\partial\theta_i\partial\theta_j = \delta_{ij}\partial f/\partial\theta_i + \sum_{m=k+1}^{p} \gamma_m\omega_{im}\omega_{jm}\Big[\sum_{n=1}^{k} \gamma_n \frac{\gamma_m + \gamma_n - 2}{\gamma_m - \gamma_n}\omega_{in}\omega_{jn} + s^{ij}\psi_i\psi_j\Big] \tag{17}$$

Approximate second order derivatives: $$\partial^2 f/\partial\theta_i\partial\theta_j \doteq \Big(\sum_{m=k+1}^{p} \omega_{im}\omega_{jm}\Big)^2 \tag{18}$$

## 4. Formulas for ML

Fitting function: $F(\Lambda,\psi) = tr(\Sigma^{-1}S) - \log|\Sigma^{-1}S| - p$ , (19)

$\Sigma = \Lambda\Lambda' + \psi^2$ ,

$\Lambda'\psi^{-2}\Lambda$ is assumed to be diagonal

Matrix of which roots and vectors are computed: $A = \psi S^{-1}\psi$

Characteristic roots: $\gamma_1 \le \gamma_2 \le \cdots \le \gamma_p$

Corresponding orthonormal vectors: $\omega_1,\omega_2,\ldots,\omega_p$

Conditional solution for $\Lambda$ for given $\psi$ : $\tilde{\Lambda} = \psi\Omega_1(\Gamma_1^{-1} - I)^{1/2}$ ,

$\Gamma_1 = \mathrm{diag}(\gamma_1,\gamma_2,\ldots,\gamma_k)$ ,

$\Omega_1 = [\omega_1\omega_2\cdots\omega_k]$

Transformation: $\psi_i = +\sqrt{e^{\theta_i}}$ , $\theta_i = \log\psi_i^2$

Function minimized by the Newton-Raphson method:

$$f(\Theta) = \sum_{m=k+1}^{p} (\log\gamma_m + 1/\gamma_m - 1) \tag{20}$$

First order derivatives: $\partial f/\partial\theta_i = \sum_{m=k+1}^{p} (1 - 1/\gamma_m)\omega_{im}^2$ (21)

Second order derivatives:

$$\partial^2 f/\partial\theta_i\partial\theta_j = -\delta_{ij}\partial f/\partial\theta_i + \sum_{m=k+1}^{p} \omega_{im}\omega_{jm}\Big[\sum_{n=1}^{k} \frac{\gamma_m + \gamma_n - 2}{\gamma_m - \gamma_n}\,\omega_{in}\omega_{jn} + \delta_{ij}\Big] \tag{22}$$

Approximate second order derivatives: $\partial^2 f/\partial\theta_i\partial\theta_j \doteq \big(\sum_{m=k+1}^{p} \omega_{im}\omega_{jm}\big)^2$ (23)

## 5. Basic Minimization Algorithm

Let $\theta$ denote a column vector with elements $\theta_1,\theta_2,\ldots,\theta_p$ (GLS and ML) or $\psi_1,\psi_2,\ldots,\psi_p$ (ULS), and let h and H denote the column vector and matrix of corresponding derivatives $\partial g/\partial\theta$ and $\partial^2 g/\partial\theta\partial\theta'$, respectively. Let $\theta^{(s)}$ denote the value of $\theta$ in the $s^{th}$ iteration and let $h^{(s)}$ and $H^{(s)}$ be the corresponding vector and matrix of first- and second-order derivatives. The iteration procedure may then be written

$$H^{(s)}\delta^{(s)} = h^{(s)} \quad , \tag{24}$$

$$\theta^{(s+1)} = \theta^{(s)} - \delta^{(s)} \quad , \tag{25}$$

where $\delta^{(s)}$ is a column vector of corrections determined by (24). The Newton-Raphson procedure is therefore easy to apply, the main computations in each iteration being the computation of the roots and vectors of A and the solution of the symmetric system (24). It has been found that the Newton-Raphson procedure is very efficient, generally requiring only a few iterations for convergence. The convergence criterion is that the largest absolute correction be less than a prescribed small number $\epsilon$. The minimizing $\theta$ may be determined very accurately, if desired, by choosing $\epsilon$ very small.

In detail, the numerical method is as follows: the starting point $\theta^{(1)}$ is chosen as (see e.g., Jöreskog, 1963, eqs. 6.20 and 7.10 or Jöreskog, 1967, eq. 26),

$$\theta_i^{(1)} = \log[(1 - k/2p)/s^{ii}] \quad , \quad \psi_i^{(1)} = +\sqrt{[(1 - k/2p)/s^{ii}]} \quad ,$$
$$\psi_i^{(1)} = .6s_{ii} \quad \text{if S is not positive definite,} \tag{26}$$

where $s_{ii}$ and $s^{ii}$ are the $i^{th}$ diagonal element of $S$ and $S^{-1}$ respectively. The exact matrix $H$ of second order derivatives given by (8), (17) or (22) may not be positive definite in the beginning. Therefore, the approximation $E$ given by (9), (18) or (23) is used in the first iteration and for as long as the maximum absolute correction is greater than a given constant $\epsilon_E$ (see sec. 7). After that, $H$ is used if it is positive definite. It has been found empirically that $E$ gives good reductions in function values in the early iterations but is comparatively ineffective near the minimum, whereas $H$ near the minimum is very effective.

In each iteration we compute the characteristic roots and vectors of $A$ by the Householder transformation to tridiagonal form, the QR method for the roots of the tridiagonal matrix and inverse iteration for the vectors. This is probably the most efficient method available (see Wilkinson, 1965). The system of equations (24) is solved by the square root factorization $H = TT'$, where $T$ is lower triangular. This shows at an early stage whether $H$ is positive definite or not.

In Heywood cases, when one or more of the $\theta_i \to -\infty$, i.e., $\psi_i \to 0$, a slight modification of the Newton-Raphson procedure is necessary to achieve fast convergence. This is due to the fact that the search for the minimum is then along a "valley" and not in a quadratic region. For ML and GLS, when $\theta_i \to -\infty$, $\partial g/\partial\theta_i \to 0$ and $\partial^2 g/\partial\theta_i\partial\theta_j \to 0$, $j = 1,2,\ldots,p$, so that when $\theta_i$ is small the $i^{th}$ element of $h$ and the $i^{th}$ row and column of $H$ and $E$ are also small. This tends to produce a "bad" correction vector $\delta$ and the function may increase instead of decrease. A simple and effective way to deal with this problem is to delete the $i^{th}$

equation in the system (24) and compute the corrections for all the other $\theta$'s from the reduced system. One then computes the correction for $\theta_i$ as

$$\delta_i = (\partial g/\partial\theta_i)/(\partial^2 g/\partial\theta_i^2) \quad . \tag{27}$$

This procedure will decrease $\theta_i$ slowly in the beginning but faster the more evident it is that $\theta_i$ is a Heywood variable. When $\theta_i$ has become less than $\log(\epsilon)$ it is not necessary to change $\theta_i$ any more unless $\partial f/\partial\theta_i$ is negative. For ULS, an analogous procedure is used. When $\psi_i$ becomes less than $\sqrt{\epsilon}$, $\psi_i$ is not changed unless $\partial f/\partial\psi_i$ is negative. Thus, the procedure corrects itself quickly if a variable is incorrectly taken as a Heywood variable.

## 6. The Program

In this section we describe briefly what the program does. Details about the input are given in section 7. For those users who feel too restricted in their choice of an input matrix, as provided by the program, the kernel of the program is available as a subroutine. The input and output parameters for that subroutine will be described in section 8.

The input data may be raw data from which the matrix to be analyzed is computed, or it may be a dispersion matrix, or it may be a correlation matrix or a correlation matrix followed by a vector of standard deviations. From these input matrices, variables may be selected to be included in the analysis, so that the matrices to be analyzed could be of smaller order than the input matrices. Variables may also be interchanged with one another. The matrices to be analyzed may be dispersion matrices or correlation matrices. The user has the option to read in a starting point for $\psi$ or have the program define a starting point (see sec. 5). This can be useful if convergence is slow and the user runs out of computer time. From the intermediate results the last $\psi$ can be read in as a new starting point and minimization can continue.

For the given matrix $S$ to be analyzed of order $p$ by $p$ and a given lower bound $k_L$ and a given upper bound $k_U$ for the number of factors, the program performs a sequence of factor analyses by the ML, ULS or GLS method of estimation chosen by the user and outlined in the previous sections. One such analysis is done for each number of factors

$$k = k_L, k_L+1, \ldots, k_U \quad .$$

The output will consist of the title with parameter listing and the matrix to be analyzed. Then for each number of factors $k$ the unrotated factor loadings, the unique variances and the varimax-rotated factor loadings are printed. For ML and GLS this is followed by $\chi^2_k$ and the corresponding degrees of freedom $d_k$, the probability level, i.e., the probability of obtaining a larger value of $\chi^2$ than that actually obtained given that the model and the assumptions hold, and Tucker and Lewis' (1970) reliability coefficient $\rho_k$, defined as follows

$$C_0 = N - 1 \ (1/6)(2p + 5)$$

$$\chi^2_0 = C_0[\sum_{i=1}^{p} \log s_{ii} - \log |S|]$$

$$d_0 = \frac{1}{2} p(p - 1)$$

$$M_0 = \chi^2_0/d_0$$

$$C_k = C_0 - (2/3)k$$

$$\chi^2_k = C_k f_{min}$$

$$d_k = \frac{1}{2} [(p - k)^2 - (p + k)]$$

$$M_k = \chi^2_k/d_k$$

$$\rho_k = \frac{M_0 - M_k}{M_0 - 1}$$

Finally the latent roots and their first differences at the minimum and the matrix of residual correlations are printed. The user also has an option to

print intermediate results consisting of the value of the function and the vector $\psi$ at each iteration. Examples of input and output can be found in Appendix B.

The following limitations are imposed on the program:

max. no. of variables after selection = 30

max. no. of variables before selection = 75

max. no. of factors = 30

storage requirements on the IBM 360/65 = 120K ( K = 1024 bytes )

The program can easily be modified to allow for a larger number of variables and factors. Instructions on how to change the maximum number of variables and factors allowed by the program can be found in Appendix C. The program is written in FORTRAN IV-G and has been tested out on the IBM 360/65 at Educational Testing Service. Double precision is used in floating point arithmetic throughout the program. With minor changes the program should run on any computer with a FORTRAN IV compiler. In computers with a single word length of 36 bits or more, single precision is probably sufficient.

Although the program has been working satisfactorily for all data analyzed so far, no claim is made that it is free of error and no warranty is given as to the accuracy and functioning of the program.

## 7. Input Data

For each set of data to be analyzed, the input consists of the following:

a. Title card

b. Parameter card

c. Data matrix

d. Selection cards (optional)

e. Starting point (optional)

f. New data or a STOP card

The function and setup of each of the above quantities are described in general terms below. Illustrative examples are given in Appendix B.

### a. Title Card

Whatever appears on this card will appear on the first page of the printed output. All 80 columns of the card are available to the user.

### b. Parameter Card

All quantities on this card, except for the logical indicators, must be punched as integers right adjusted within the field.

| | |
|---|---|
| cols. 1-5 | number of observations N |
| cols. 6-10 | order of data matrix ( $p_0$ ), before selection of variables |
| cols. 11-15 | lower bound for the number of factors $k_L$ |
| cols. 16-20 | upper bound for the number of factors $k_U$ |
| cols. 21-25 | maximum number of iterations allowed for each number of factors k |
| col. 31 | logical variable which determines whether selection of variables from the data matrix is desired<br>col. 31 = T, if selection of variables is wanted<br>col. 31 = F, if no selection of variables is wanted |

col. 32 logical variable which determines whether a dispersion matrix or a correlation matrix is to be analyzed

col. 32 = T, if a dispersion matrix is to be analyzed

col. 32 = F, if a correlation matrix is to be analyzed

col. 41 integer indicator which determines whether raw data, a dispersion matrix, a correlation matrix or a correlation matrix with standard deviations are read in to determine the matrix to be analyzed

col. 41 = 1, read in raw data

col. 41 = 2, read in a dispersion matrix

col. 41 = 3, read in a correlation matrix (followed by a vector of standard deviations if col. 32 is T)

col. 42 integer indicator which determines which method of estimation is to be used

col. 42 = 1 for ULS

col. 42 = 2 for GLS

col. 42 = 3 for ML

col. 43 integer indicator which determines whether intermediate results are to be printed

col. 43 = 0, if no intermediate results are to be printed

col. 43 = 1, if intermediate results are to be printed (see sec. 6)

col. 44 — integer indicator which determines whether a starting point is defined by the program or is to be supplied by the user (see secs. 5 and 6)

col. 44 = 0 , if a starting point is defined by the program

col. 44 = 1 , if a starting point is read in as data

cols. 46-55 — convergence criterion $\epsilon$ (see sec. 5). For reasonable results use $\epsilon \leq .005$ .

cols. 56-65 — $\epsilon_E$ ; if all elements of the correction vector are less than $\epsilon_E$ the <u>exact</u> second order derivatives are computed in the minimization algorithm, otherwise the <u>approximate</u> second order derivatives are used (see sec. 5). From our experience $\epsilon_E = .1$ seems reasonable.

cols. 66-70 — logical tape (disk) number of <u>scratch tape</u> (disk) used for intermediate storage

<u>c. Data Matrix</u>

The data matrix is preceded by a format card, containing at most 80 columns, beginning with a left parenthesis and ending with a right parenthesis. The format must specify floating point numbers consistent with the way in which the elements of the matrix are punched. Users who are unfamiliar with FORTRAN are referred to a FORTRAN Manual where format rules are given.

The input matrix can be any one of the following:

If col. 41 = 1 on the parameter card an N x p matrix of raw data is read in, one row at a time, starting a new card for each row. The matrix is preceded by a format card as described above.

If col. 41 = 2 the lower triangular part of a dispersion matrix, including the diagonal, is read in. The matrix should be punched row-wise as one long vector, i.e., there is no need to go to a new card if a new row starts. Again the matrix should be preceded by a format card.

If col. 41 = 3 and col. 32 = F the lower triangular part of a correlation matrix, including the diagonal, is read in. The matrix should be punched row-wise as one long vector, and should be preceded by a format card.

If col. 41 = 3 and col. 32 = T the lower triangular part of a correlation matrix including the diagonal, is read in. The matrix should be punched row-wise as one long vector, and should be preceded by a format card. This matrix is then followed by a format card and a row-vector of standard deviations.

d. Selection Cards (optional)

Omit if column 31 of the parameter card is F. Otherwise the first card will have an integer value p punched in columns 1-5, right adjusted within the field. This integer will specify the new order of the data matrix after selection of variables ( $p \leq p_0$ ).

The next card will contain integers, right adjusted in five column fields (i.e., sixteen such values will fit on one card), specifying which columns (rows) are to be included. For example, if $p_0 = 6$ , $p = 3$ and the first, second and fifth columns (rows) are to be excluded, this card

would have a 3 punched in column 5, a 4 punched in column 10 and a 6 punched in column 15.

Note that if $p = p_0$ there will be no reduction in the size of the data matrix but columns (rows) can be interchanged.

e. Starting Point (optional)

Omit if column 44 on the parameter card is zero. Otherwise read in a starting $\psi$ vector punched according to a format of 5D15.7 (see sec. 6) for each number of factors $k$.

f. Stacked Data

In the preceding paragraphs we have described how each set of data should be set up. Any number of such sets of data may be stacked together and analyzed in one run. After the last set of data in the stack, there must be a card with the word STOP punched in columns 1-4.

## 8. Subroutine NWTRAP

As an alternative to the program the user can write his own main program in which he calls the minimization package with a CALL to NWTRAP. The following subroutines are called by NWTRAP and thus are part of the package: FCTGR, INCPSI, SØLVE, ISMSL, HFWLIN, TRIDI, EIGVEC and QRB.

The minimization package should be called with the following sequence of FØRTRAN statements:

```
      DØ 10 K = KL,KU
      CALL NWTRAP(P,K,I1,I2,I3,S,EPS,EPSE,MAXIT,A,E,X,Y,FO,DET)
      .
      .
   10 CØNTINUE
```

The DØ loop runs from KL, the lower bound on the number of factors to KU, the upper bound on the number of factors.

Next follows a description of the parameters of NWTRAP:

Input Parameters

| | |
|---|---|
| P | order of data matrix S |
| K | number of factors |
| I1 | determines which method of estimation is to be used |
| | I1 = 1 for ULS |
| | I1 = 2 for GLS |
| | I1 = 3 for ML |
| I2 | determines whether intermediate results are to be printed (see sec. 6) |
| | I2 = 0 , if no intermediate results are to be printed |
| | I2 = 1 , if intermediate results are to be printed |

I3 — determines whether the starting point is defined by the program or read in by the user as data (see sec. 6)

I3 = 0 , if the starting point is defined by the program (see sec. 5)

I3 = 1 , a starting vector $\psi$ is read in as data with a format of 5D15.7 for each number of factors K

S — data matrix, stored row-wise as a vector. Should be singly dimensioned in the calling program by at least $(P(P + 1))/2$ .

EPS — convergence criterion $\epsilon$ (see sec. 5). For reasonable results use $\epsilon \leq .005$ .

EPSE — if all elements of the correction vector are less than EPSE, the exact second order derivatives are used in the minimization algorithm, otherwise the approximate second order derivatives are used. From our experience EPSE = .1 seems reasonable.

MAXIT — maximum number of iterations allowed for each number of factors K . Program exits if this number is exceeded.

Output Parameters

A — matrix of unrotated factor loadings, stored row-wise as a vector. Should be singly subscripted in the calling program by at least P x K .

| | |
|---|---|
| E | dummy vector. Should be dimensioned in the calling program by at least $(P \times (P + 1))/2$ . |
| X | vector of unique variances. Should be dimensioned in the calling program by at least $P$ . |
| Y | vector of latent roots of $A$ at the minimum. Should be dimensioned in the calling program by at least $P$ . |
| FO | the value of the function at the minimum |
| DET | the determinant of the data matrix $S$ |

## References

Clarke, M. R. B. A rapidly convergent method for maximum-likelihood factor analysis. The British Journal of Mathematical and Statistical Psychology, 1970, 23, 43-52.

Harman, H. H. Modern factor analysis. (2nd ed.) Chicago: University of Chicago, 1967.

Jöreskog, K. G. Statistical estimation in factor analysis. Stockholm: Almqvist and Wiksell, 1963.

Jöreskog, K. G. Some contributions to maximum likelihood factor analysis. Psychometrika, 1967, 32, 443-482. (a)

Jöreskog, K. G. UMLFA--A computer program for unrestricted maximum likelihood factor analysis. Research Memorandum 66-20. Princeton, N.J.: Educational Testing Service. Revised edition, 1967. (b)

Jöreskog, K. G. and Goldberger, A. S. Factor analysis by generalized least squares. Research Bulletin 71-26. Princeton, N.J.: Educational Testing Service, 1971.

Tucker, L. R and Lewis, C. A reliability coefficient for maximum likelihood factor analysis. In L. R Tucker, W. D. Love and C. Lewis, Topics in factor analysis and ONR Technical Report, Contract US NAVY/00014-67-A-0303-0003. Champaign, Ill.: University of Illinois at Urbana-Champaign, 1970. Pp. 1-18.

Wilkinson, J. H. The algebraic eigenvalue problem. Oxford: Oxford University Press, 1965.

# Appendix A

## Listing of the FORTRAN Program

```
C     PROGRAM UFABY3
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      REAL HEAD,HALT
      COMMON/LIT/OS,OR
      COMMON/MN/DF,DET,FO,P,K,KL,N,NT,IND,P2
      DIMENSION FMT(10),S(2850),A(2850),HEAD(20),YY(30),E(465),Y(30)
      DATA HALT/4HSTOP/,FMT/24H(5X,I5,5X,10F11.3)       /
      CALL ERRSET(208,256,-1,1)
    1 READ(5,100)HEAD
      IF(HEAD(1).EQ.HALT)CALL EXIT
      READ(5,200)N,P,KL,KU,MAXIT  ,OS,OR,INDA,IND,IO,IS,EPS,EPSE,NT
      WRITE(6,300)HEAD,N,P,KL,KU,MAXIT,OS,OR,INDA,IND,IO,IS,EPS,EPSE,NT
      P2=(P*(P+1))/2
      CALL REX(P,P2,N,S,A,INDA)
      CALL PMSL(P,P,S,FMT,24HMATRIX TO BE ANALYZED   ,1,6,1)
      GO TO (4,2,2),IND
    2 REWIND NT
      WRITE(NT)(S(I),I=1,P2)
    4 DO 5 I=KL,KU
      K=I
      DF=((P-K)**2-(P+K))/2
      IF(DF.LT.0)GO TO 10
      CALL NWTRAP(P,K,   IND,IO,IS,S,EPS,EPSE,MAXIT,A,E,YY,Y,FO,DET)
      CALL FINOT(YY,Y,A,E,S)
    5 CONTINUE
      GO TO 1
   10 WRITE(6,400)K
      GO TO 1
  100 FORMAT(20A4)
  200 FORMAT(5I5,5X,2L1,8X,4I1,1X,2F10.0,I5)
  300 FORMAT(1H1,20A4/'0N=',I5/'0P=',I5/'0KL=',I5/'0KU=',I5/'0MAXIT=',
     1I5/'0LOGICAL VARIABLES= ',2L1/'0INTEGER VARIABLES= ',4I1/'0EPS=',
     2F10.7/'0EPSE=',F10.7/'0LOGICAL SCRATCH TAPE (DISK) NUMBER=',I3)
  400 FORMAT(1H0,'ERROR EXIT - DEGREES OF FREEDOM FOR ',I3,' FACTORS IS
     1NEGATIVE')
      END
      SUBROUTINE REX(P,P2,N,S,E,IND)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/LIT/OS,OR
      DIMENSION S(1),E(1),Y(100),X(100),FMT(10)
C ******* DEFINE INPUT MATRIX (RAW DATA OR S OR R OR R AND ST.DEV.)
      READ(5,100) FMT
      GO TO (5,25,30),IND
C ******* COMPUTE S FROM RAW DATA
    5 L=0
      DO 10 I=1,P
      X(I)=0.DO
      DO 10 J=1,I
      L=L+1
      S(L)=0.DO
   10 CONTINUE
      DO 15 K=1,N
      READ(5,FMT)(Y(J),J=1,P)
      L=0
      DO 15 I=1,P
      X(I)=X(I)+Y(I)
      DO 15 J=1,I
      L=L+1
      S(L)=S(L)+Y(I)*Y(J)
```

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23

```
   15 CONTINUE
      FN=1.DO/N
      L=0
      DO 20 I=1,P
      X(I)=X(I)*FN
      DO 20 J=1,I
      L=L+1
      S(L)=S(L)*FN-X(I)*X(J)
   20 CONTINUE
      IF(OR)GO TO 50
   21 L=0
      DO 22 I=1,P
      L=L+I
   22 Y(I)=1.DO/DSQRT(S(L))
      GO TO 35
C ******* READ IN S
   25 READ(5,FMT)(S(I),I=1,P2)
      IF(OR)GO TO 50
      GO TO 21
C ******* READ IN R
   30 READ(5,FMT)(S(I),I=1,P2)
      IF(.NOT.OR)GO TO 50
C ******* READ IN STANDARD DEVIATIONS
      READ(5,100)FMT
      READ(5,FMT)(Y(I),I=1,P)
   35 L=0
      DO 40 I=1,P
      DO 40 J=1,I
      L=L+1
      S(L)=S(L)*Y(I)*Y(J)
   40 CONTINUE
   50 IF(OS)CALL SELECT(P,P2,S,E)
  100 FORMAT(10A8)
      RETURN
      END
      SUBROUTINE SELECT(P,P2,S,E)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      DIMENSION S(1),E(1),MM(100)
      DO 5 I=1,P2
      E(I)=S(I)
    5 CONTINUE
      READ(5,100)P
      READ(5,200)(MM(I),I=1,P)
      P2=(P*(P+1))/2
      L=0
      DO 15 I=1,P
      DO 15 J=1,I
      LI=MM(I)
      L=L+1
      LJ=MM(J)
      IF(LI.GT.LJ)GO TO 10
      LI=LJ
      LJ=MM(I)
   10 IJ=((LI-1)*LI)/2+LJ
      S(L)=E(IJ)
   15 CONTINUE
  100 FORMAT(I5)
  200 FORMAT(16I5)
```



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```
      RETURN
      END
      SUBROUTINE FINOT(V,Y,A,E,S)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/LIT/OS,OR
      COMMON/MN/DF,DET,FO,P,K,KL,N,NT,IND,P2
      DIMENSION V(1),E(1),FMT(10),Y(1),S(1),A(1)
      DATA BLANK/8H  - -   /
      DATA FMT/24H(5X,I5,5X,10F11.3)      /
      GO TO (1,2,3),IND
    1 WRITE(6,100)K
      GO TO 6
    2 WRITE(6,200)K
      GO TO 4
    3 WRITE(6,300)K
    4 REWIND NT
      READ(NT)(S(I),I=1,P2)
    6 CALL PMSL(P,K,A,FMT,28HUNROTATED FACTOR LOADINGS   ,0,7,0)
      WRITE(6,400)(V(I),I=1,P)
      CALL VARMAX(P,K,A,V,S)
      CALL PMSL(P,K,A,FMT,32HVARIMAX-ROTATED FACTOR LOADINGS ,0,8,0)
      GO TO (25,5,5),IND
    5 IF(K.NE.KL)GO TO 15
      CO=N-1.DO-(2.DO*P+5.DO)/6.DO
      FMO=0.DO
      IF(.NOT.ORIGO TO 9
      L=0
      DO 8 I=1,P
      L=L+I
      FMO=FMO+DLOG(S(L))
    8 CONTINUE
    9 FMO=CO*(FMO-DLOG(DET))/(.5DO*P*(P-1.DO))
   15 CHSQ=(CO-2.DO*K/3.DO)*FO
      PROB=CHIPR(DF,CHSQ)
      RHOK=(FMO-CHSQ/DF)/(FMO-1.DO)
      NF=DF
      WRITE(6,500)NF,CHSQ,PROB,RHOK
      L=P
      DO 22 I=1,P
      V(I)=Y(L)
   22 L=L-1
      DO 23 I=1,P
      Y(I)=V(I)
   23 CONTINUE
   25 V(1)=BLANK
      DO 28 I=2,P
      V(I)=Y(I-1)-Y(I)
   28 CONTINUE
      WRITE(6,600)Y(1),V(1),(I,Y(I),V(I),I=2,P)
C *** COMPUTE RESIDUAL CORRELATIONS   ( S - LAMBDA*LAMBDA' )
      L=0
      DO 35 I=1,P
      I1=(I-1)*K
      JL=0
      DO 30 J=1,I
      L=L+1
      RHOK=0.DO
      DO 29 M=1,K
```

```
      IL=I1+M
      JL=JL+1
   29 RHOK=RHOK+A(IL)*A(JL)
      E(L)=S(L)-RHOK
   30 CONTINUE
      IF(E(L).LE.0.D0)E(L)=1.D0
      V(I)=1.D0/DSQRT(E(L))
   35 CONTINUE
      L=0
      DO 40 I=1,P
      DO 40 J=1,I
      L=L+1
   40 E(L)=E(L)*V(I)*V(J)
      CALL PMSL(P,P,E,FMT,24HRESIDUAL CORRELATIONS    ,1,6,1)
  100 FORMAT('1UNWEIGHTED LEAST SQUARES SOLUTION FOR ',I3,' FACTORS')
  200 FORMAT('1GENERALIZED LEAST SQUARES SOLUTION FOR ',I3,' FACTORS')
  300 FORMAT('1MAXIMUM LIKELIHOOD SOLUTION FOR ',I3,' FACTORS')
  400 FORMAT(1HO,10X,'UNIQUE VARIANCES'/1HO,(10F11.3))
  500 FORMAT(1HO,10X,'CHISQUARE WITH',I5,  ' DEGREES OF FREEDOM IS',
     1F16.4//11X,'PROBABILITY LEVEL IS ',F6.3//11X,34HTUCKER'S RELIABILI
     2TY COEFFICIENT= ,F6.3)
  600 FORMAT(1H1,15X,'LATENT ROOTS',5X,'FIRST DIFFERENCES'// 10X,'1',
     1D16.4,11X,A8/(6X,I5,D16.4,D19.4))
      RETURN
      END
      SUBROUTINE VARMAX(NT,NF,A,X,S)
C     ORTHOGONAL ROTATION BY KAISER'S VARIMAX METHOD
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),X(1),S(1)
      AT=NT
      AF=NF
      CK=DSQRT(2.D0)*.5D0
      L=0
      I1=0
      DO 16 I=1,NT
      I1=I1+I
      X(I)=1.D0/DSQRT(S(I1)-X(I))
      DO 16 J=1,NF
      L=L+1
      A(L)=A(L)*X(I)
   16 CONTINUE
      NF1=NF-1
   22 DO 90 K1=1,NF1
      K11=K1+1
      DO 90 K2=K11,NF
      AA=0.D0
      BB=0.D0
      CC=0.D0
      DD=0.D0
      IK1=K1
      IK2=K2
      DO 40 I=1,NT
      U=(A(IK1)+A(IK2))*(A(IK1)-A(IK2))
      V=2.D0*A(IK1)*A(IK2)
      AA=AA+U
      BB=BB+V
      CC=CC+(U+V)*(U-V)
      DD=DD+U*V
      IK1=IK1+NF
```

```
      IK2=IK2+NF
   40 CONTINUE
      XNUM=2.D0*(DD-AA*BB/AT)
      XDEN=CC-(AA**2-BB**2)/AT
      ANUM=DABS(XNUM)
      ADEN=DABS(XDEN)
      IF(ANUM-ADEN)47,46,52
   46 IF(ANUM)57,90,57
   47 ATA=ANUM/ADEN
      IF(ATA-.00116D0)75,49,49
   49 ACO=1.D0/DSQRT(1.D0+ATA**2)
      ASI=ATA*ACO
      GO TO 59
   52 ACOT=ADEN/ANUM
      IF(ACOT-.00116D0)81,54,54
   54 ASI=1.D0/DSQRT(1.D0+ACOT**2)
      ACO=ACOT*ASI
      GO TO 59
   57 ACO=CK
      ASI=CK
   59 ACO=DSQRT((1.D0+ACO)*.5D0)
      ASI=ASI/(2.D0*ACO)
      ACO=DSQRT((1.D0+ACO)*.5D0)
      ASI=ASI/(2.D0*ACO)
      IF(XDEN)64,64,68
   64 XCO=ACO*CK
      XSI=ASI*CK
      ACO=XCO+XSI
      ASI=XCO-XSI
   68 CO=ACO
      IF(XNUM)72,72,69
   69 SI=ASI
      GO TO 84
   72 SI=-ASI
      GO TO 84
   75 IF(XDEN)76,79,79
   76 CO=CK
      SI=CK
      GO TO 84
   79 K=K-1
      GO TO 90
   81 ACO=0.D0
      ASI=1.D0
      GO TO 59
   84 K=(NF*(NF-1))/2
      IK1=K1
      IK2=K2
      DO 89 I=1,NT
      AA=A(IK1)
      BB=A(IK2)
      A(IK1)=AA*CO+BB*SI
      A(IK2)=-AA*SI+BB*CO
      IK1=IK1+NF
   89 IK2=IK2+NF
   90 CONTINUE
   92 IF(K)93,93,22
   93 L=0
      DO 95 I=1,NT
      X(I)=1.D0/X(I)
```

```
      DO 95 J=1,NF
      L=L+1
      A(L)=A(L)*X(I)
   95 CONTINUE
C ***** CHANGE SIGN OF COLUMNS WHERE WARRANTED
      DO 130 J=1,NF
      AA=0.DO
      IJ=J
      DO 110 I=1,NT
      BB=DABS(A(IJ))
      IF(BB.LE.AA) GO TO 110
      AA=BB
      CC=A(IJ)
      IJ=IJ+NF
  110 CONTINUE
      IF(CC.GE.0.DO)GO TO 130
      IJ=J
      DO 120 I=1,NT
      A(IJ)=-A(IJ)
  120 IJ=IJ+NF
  130 CONTINUE
      RETURN
      END
      END
      FUNCTION CHIPR(DF,CHSQ)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      A=.5*DF
      X=.5*CHSQ
      IF(X .GT. 0.) GO TO 100
      CHIPR=1.
      GO TO 170
  100 TERM=1.
      SUM=0.
      COFN=A
      IF(13.-X) 110,110,120
  110 IF(A-X) 140,140,120
  120 CON=1.
      FACT=-A
  130 TEMP=SUM
      SUM=SUM+TERM
      COFN=COFN+1.
      TERM=TERM*X/COFN
      IF(SUM-TEMP) 160,160,130
  140 CON=0.
      FACT=X
  150 TEMP=SUM
      SUM=SUM+TERM
      COFN=COFN-1.
      RATIO=COFN/X
      TERM=TERM*RATIO
      IF(SUM-TEMP) 160,160,150
  160 CHIPR=CON+DEXP(DLOG(SUM)-X+A*DLOG(X)-DLGAMA(A))/FACT
  170 RETURN
      END
      END
      SUBROUTINE NWTRAP(P,K,   IND,IO,IS,S,EPS,EPSE,MAXIT,A,E,YY,Y,FO,
     1DET)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
```

```
      COMMON/KERN/G(30),V(30),VB(30)       ,D2(30),S1(30),S2(30),S3(30),
     1EPSU,BND,IM(30),MOR,KP,MORE,MAXTRY,P2,KP1
      DIMENSION S(1),E(1),A(1),YY(1),Y(1)
      IF(IO.EQ.1)WRITE(6,300)
      MAXTRY=10
      EPSU=.0500
      MORE=0
      KP=P-K
      KP1=KP+1
      IF(EPS.LE..005000001)GO TO 1
      EPS=.005
      WRITE(6,100)
    1 P2=(P*(P+1))/2
      GO TO (4,2,2),IND
    2 CALL ISMSL (P,S,S,Y,DET,.5D-12,IERR)
      BND=DLOG(EPS)
      IF(IERR.NE.0)CALL EXIT
C******GENERATE STARTING PSI
    4 F0=1.D10
      GO TO (5,10,10),IND
    5 CALL ISMSL (P,S,E,Y,DET,.5D-12,IERR)
      BND=DSQRT(EPS)
      IF(IERR.EQ.0)GO TO 10
      IF(IS.EQ.0)GO TO 7
    3 READ(5,500)(V(I),I=1,P)
      GO TO (15,12,12),IND
   12 DO 13 I=1,P
   13 V(I)=2.D0*DLOG(V(I))
      GO TO 15
    7 L=0
      DO 6 I=1,P
      L=L+I
      V(I)=.6*S(L)
    6 CONTINUE
      GO TO 15
   10 IF(IS.EQ.1)GO TO 3
      FT=1.D0-K/(2.D0*P)
      L=0
      DO 11 I=1,P
      L=L+I
      GO TO (8,9,9),IND
    8 V(I)=DSQRT(FT/E(L))
      GO TO 11
    9 V(I)=DLOG(FT/S(L))
   11 CONTINUE
   15 MOR=1
      ITER=0
   16 CALL FCTGR(P,K,S,A,E,YY,Y,F0,IND)
      ITER=ITER+1
      IF(IO.EQ.0)GO TO 25
      GO TO (20,22,22),IND
   20 WRITE(6,200)ITER,F0,(V(I),I=1,P)
      GO TO 25
   22 WRITE(6,200)ITER,F0,(YY(I),I=1,P)
   25 IF(ITER.LE.MAXIT  )GO TO 26
      WRITE(6,600)
      CALL EXIT
   26 IF (MOR.EQ.0) GO TO 45
C******COMPUTE NEW PSI
```

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```
      CALL INCPSI(P,K,IND,A,E,S,YY,Y,EPSE,EPS)
      GO TO 16
C******COMPUTE LAMBDA AT THE MINIMUM
   45 L=0
      GO TO (50,55,55),IND
   50 DO 52 I=1,P
      DO 51 J=1,I
      L=L+1
   51 E(L)=S(L)
   52 E(L)=E(L)-YY(I)
      L=1
      GO TO 61
   55 DO 60 I=1,P
      DO 60 J=1,I
      L=L+1
      E(L)=YY(I)*YY(J)*S(L)
   60 CONTINUE
      L=-1
   61 CALL HFWLIN(P,P2, L,K,E,Y,A,G ,VB,S1,S2,S3)
   62 L=0
      DO 65 I=1,P
      DO 65 J=1,K
      L=L+1
      GO TO (63,64,64),IND
   63 A(L )=A(L )*DSQRT(Y(J))
      GO TO 65
   64 A(L )=YY(I)*A(L )*DSQRT(1.D0/Y(J)-1.D0)
   65 CONTINUE
      GO TO (80,66,66),IND
   66 DO 75 I=1,P
      YY(I)=YY(I)**2
   75 CONTINUE
   80 RETURN
  100 FORMAT(1H0,'SPECIFIED CONVERGENCE CRITERION TOO LARGE  - EPS SET E
     1QUAL TO .005 ( SEE WRITE-UP)')
  200 FORMAT(1H0,'ITER=',I4,5X,'F=     ',D15.7/15X,'PSI=  ',7D15.7/
     1(21X,7D15.7))
  300 FORMAT(1H1,'INTERMEDIATE RESULTS'/)
  500 FORMAT(5D15.7)
  600 FORMAT('0MAXIMUM NUMBER OF ITERATIONS EXCEEDED')
      END
      SUBROUTINE FCTGR(P,K,S,A,E,YY,Y,F0,IND)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/KERN/G(30),V(30),VB(30)      ,D2(30),S1(30),S2(30),S3(30),
     1EPSU,BND,IM(30),MOR,KP,MORE,MAXTRY,P2,KP1
      DIMENSION S(1),A(1),YY(1),Y(1),E(1)
      ITRY=0
C******CHOOSE METHOD OF ESTIMATION(I.E.ULS,GLS.OR,ML)
    2 GO TO (1,30,30),IND
C******DEFINE(S-PSI**2),STORED IN E
    1 DO 5 I=1,P2
      E(I)=S(I)
    5 CONTINUE
      L=0
      DO 10 I=1,P
      YY(I)=V(I)**2
      L=L+I
      E(L)=E(L)-YY(I)
```

```
   10 CONTINUE
C******COMPUTE ROOTS AND VECTORS OF (S-PSI**2)
      L=P
      IF(MORE.EQ.0)L=KP
      CALL HFWLIN (P,P2,-1,L ,E,Y,A,G ,D2,S1,S2,S3)
      F=0.D0
      DO 15 I=1,KP
      F=F+Y(I)**2
   15 CONTINUE
      F=F* .5D0
      JJ=0
      DO 25 I=1,P
      SUM=0.D0
      DO 20 J=1,KP
      JJ=JJ+1
      SUM=SUM+Y(J)*A(JJ)**2
   20 CONTINUE
      G(I)=-2.D0*V(I)*SUM
      JJ=JJ+MORE
   25 CONTINUE
      GO TO 65
C******DEFINE (PSI*S**-1*PSI), STORED IN E
   30 L=0
      DO 35 I=1,P
      YY(I)=DSQRT(DEXP(V(I)))
      DO 35 J=1,I
      L=L+1
      E(L)=YY(I)*YY(J)*S(L)
   35 CONTINUE
C******COMPUTE ROOTS AND VECTORS OF (PSI*S**-1*PSI)
      L=P
      IF(MORE.EQ.0)L=KP
      CALL HFWLIN (P,P2,1,L,E,Y,A,G ,D2,S1,S2,S3)
      F=0.D0
      GO TO (85,36,45),IND
   36 DO 40 I=1,KP
      F=F+(Y(I)-1.D0)**2
      D2(I)=Y(I)**2-Y(I)
   40 CONTINUE
      F=F*.5D0
      GO TO 55
   45 DO 50 I=1,KP
      F=F+1.D0/Y(I)+DLOG(Y(I))
      D2(I)=1.D0-1.D0/Y(I)
   50 CONTINUE
      F=F-KP
C******COMPUTE FIRST ORDER DERIVATIVES
   55 JJ=0
      DO 60 I=1,P
      SUM=0.D0
      DO 58 J=1,KP
      JJ=JJ+1
      SUM=SUM+D2(J)*A(JJ)**2
   58 CONTINUE
      G(I)=SUM
      JJ=JJ+MORE
   60 CONTINUE
   65 IF(F0-F.LT.0.D0) GO TO 72
      F0=F
```

```
      DO 66 I=1,P
      VB(I)=V(I)
   66 CONTINUE
      RETURN
   72 ITRY=ITRY+1
      IF(ITRY.LE.MAXTRY)GO TO 73
      ITER=ITER-1
      WRITE(6,100)ITER,F,I,FO,MAXTRY
      CALL EXIT
   73 DO 75 I=1,P
      V(I)=(V(I)+VB(I))*.5D0
   75 CONTINUE
      GO TO 2
   85 CALL EXIT
  100 FORMAT('0F(',I3,')=',D15.7,' IS GREATER THAN F(',I3,')=',D15.7,' E
     1VEN AFTER',I4,' SUCCESSIVE HALVINGS OF THE INTERVAL'/ '0INCREASING
     2 MAXTRY IN SUBROUTINE NWTRAP MIGHT SOLVE THE PROBLEM')
      END
      SUBROUTINE INCPSI(P,K,IND,A,E,S,YY,Y,EPSE,EPS)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/KERN/G(30),V(30),VB(30)       ,D2(30),S1(30),S2(30),S3(30),
     1EPSU,BND,IM(30),MOR,KP,MORE,MAXTRY,P2,KP1
      DIMENSION E(1),A(1),S(1),YY(1),Y(1)
C ******* COMPUTE EXACT SECOND ORDER DERIVATIVES
      KPP=KP
      IF(MORE.EQ.0)GO TO 80
      KPP=P
      DO 1 I=1,P2
    1 E(I)=0.D0
      GO TO (11,5,11),IND
C ******* COMPUTE EXACT SECOND ORDER DERIVATIVES
    5 DO 10 I=1,P
      LL=I
      S2(I)=DSQRT(Y(I))
      DO 10 J=1,P
      A(LL)=A(LL)*S2(I)
      LL=LL+P
   10 CONTINUE
   11 DO 20 M=1,KP
      T1=Y(M)
      T2=.5D0*T1
      GO TO (13,12,12),IND
   12 T1=T1  -2.D0
   13 DO 14 N=KP1,P
      S3(N)=(T1+Y(N))/(Y(M)-Y(N))
   14 CONTINUE
      L=0
      LL=M
      DO 20 I=1,P
      LJ=M
      I1=(I-1)*P
      DO 18J=1,I
      J1=(J-1)*P
      L=L+1
      SUM=0.D0
      DO 15 N=KP1,P
      IN=N+I1
      JN=N+J1
```

```
      SUM=SUM+S3(N)*A(IN)*A(JN)
   15 CONTINUE
      GO TO (17,23,16),IND
   16 E(L)=E(L)+A(LL)*A(LJ)*SUM
      GO TO 24
   17 E(L)=E(L)+A(LL)*A(LJ)*SUM *V(I)*V(J)
      GO TO 24
   23 E(L)=E(L)+A(LL)*A(LJ)*(SUM+YY(I)*YY(J)*S(L))
   24 LJ=LJ+P
   18 CONTINUE
      GO TO (22,21,19),IND
   22 E(L)=E(L)+A(LL)**2*(YY(I)-T2)
      GO TO 21
   19 E(L)=E(L)+A(LL)**2
   21 LL=LL+P
   20 CONTINUE
      L=0
      DO 27 I=1,P
   27 S3(I)=G(I)
      GO TO (25,35,40),IND
   25 DO 30 I=1,P
      DO 30 J=1,I
      L=L+1
   26 E(L)=E(L)*4.D0
   30 CONTINUE
      GO TO 50
   35 DO 38 I=1,P
      L=L+I
   38 E(L)=E(L)+G(I)
      GO TO 50
   40 DO 45 I=1,P
      L=L+I
   45 E(L)=E(L)-G(I)
   50 L=0
      IHM=0
      DO 75 I=1,P
      L=L+I
      IF(E(L).GT.EPSU )GO TO 75
      IHM=IHM+1
      S1(IHM)=E(L)
      S2(IHM)=G(I)
      IM(IHM)=I
      E(L)=1.D0
      I1=I-1
      IF(I1.EQ.0)GO TO 73
      DO 72 J=1,I1
      JN=L-J
   72 E(JN)=0.D0
   73 JN=L
      I1=P-1
      DO 74 J=I,I1
      JN=JN+J
   74 E(JN)=0.D0
   75 CONTINUE
      CALL SOLVE(P,E,G,.5D-12,IERR)
      IF(MORE.EQ.0) GO TO 95
      IF(IERR.EQ.0)GO TO 101
      DO 76 I=1,P
   76 G(I)=S3(I)
```

```
C ******* COMPUTE EXPECTED SECOND ORDER DERIVATIVES
   80 L=0
      DO 92 I=1,P
      I1=(I-1)*KPP
      DO 92 J=1,I
      J1=(J-1)*KPP
      SUM=0.DO
      DO 90 M=1,KP
      IN=M+I1
      JN=M+J1
      SUM=SUM+A(IN)*A(JN)
   90 CONTINUE
      L=L+1
      E(L)=SUM**2
   92 CONTINUE
      GO TO (93,50,50),IND
   93 L=0
      DO 94 I=1,P
      DO 94 J=1,I
      L=L+1
      E(L)=E(L)* V(I)* V(J)*4.DO
   94 CONTINUE
      GO TO 50
   95 IF(IERR.EQ.0)GO TO 101
      WRITE(6,500)
      CALL EXIT
  101 IF(IHM.EQ.0)GO TO 87
      DO 102 I=1,IHM
      L=IM(I)
      G(L)=0.DO
      IF(S1(I).LT.1.D-10)GO TO 102
      G(L)=S2(I)/S1(I)
  102 CONTINUE
   87 DO 97 I=1,P
      V(I)=VB(I)-G(I)
   97 CONTINUE
      DO 98 I=1,P
      IF(V(I).GT.BND   )GO TO 96
      V(I)=BND
      GO TO 98
   96 IF(DABS(G(I)).LT.EPSE)GO TO 98
      MORE=0
      RETURN
   98 CONTINUE
C******TEST FOR CONVERGENCE
      DO 105 I=1,P
      IF(V(I).LE.BND   )GO TO 105
      IF(DABS(G(I)).GT.EPS)GO TO 106
  105 CONTINUE
      MOR=0
      RETURN
  106 MORE=K
      RETURN
  500 FORMAT('0EXPECTED SECOND ORDER DERIVATIVES MATRIX IS NOT POSITIVE
     1DEFINITE'/' THIS SHOULD NEVER HAPPEN - CHECK YOUR INPUT DATA')
      END
      SUBROUTINE ISMSL(N,A,B,Y,D,EPS,IERR)
C ***** INVERT SYMMETRIC MATRIX STORED LINEARLY
C         N    ORDER OF MATRIX
```

```
C          A     MATRIX TO BE INVERTED,STORED LINEARLY,MUST BE GRAMIAN
C          B     A INVERSE,STORED AS A VECTOR
C          Y     INTERNAL DUMMY ARRAY,MUST BE DIMENSIONED IN CALLING PROGRAM BY N
C          D     DETERMINANT(A)
C          EPS   IF ANY PIVOTAL ELEMENT IS LESS THAN EPS,A IS CONSIDERED SINGULAR
C                AND CONTROL IS TRANSFERRED TO THE CALLING PROGRAM WITH IERR=1
C          IERR =0 IMPLIES A IS NON-SINGULAR
C **********************
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      DIMENSION A(1),B(1),Y(1)
      IERR=0
      NN=(N*(N+1))/2
      DO 5 I=1,NN
    5 B(I)=A(I)
      D=1.D0
      IF(N.EQ.1) GO TO 260
      DO 240 L=1,N
      F=B(1)
      IF(F.LT.EPS) GO TO 700
      D=D*F
      F=1.D0/F
      NA=1
      DO 210 I=1,N
      NA=NA+I-1
  210 Y(I)=B(NA)
      NA=0
      NB=1
      DO 220 I=2,N
      NB=NB+1
      H=Y(I)*F
      DO 220 J=2,I
      NB=NB+1
      NA=NA+1
  220 B(NA)=B(NB)-Y(J)*H
      DO 230 J=2,N
      NA=NA+1
  230 B(NA)=-Y(J)*F
  240 B(NN)=-F
      DO 250 I=1,NN
  250 B(I)=-B(I)
      RETURN
  260 L=1
      F=B(1)
      IF(F.LT.EPS) GO TO 700
      B(1)=1.D0/F
      D=F
      RETURN
  700 WRITE (6,1) L,F,D
      IERR=1
      RETURN
    1 FORMAT('0MATRIX IS NOT POSITIVE DEFINITE',I5,2D15.7)
      SUBROUTINE SOLVE(N,A,X,ERR,IND)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),X(1)
      IND=0
      IF(N.EQ.1)GO TO 4
      DO 2 J=2,N
      J1=J-1
      JD=J1+(J1*(J1-1))/2
```

```
      IF(A(JD).LT.ERR)GO TO 5
      Y=1.DO/A(JD)
      X(J1)=X(J1)*Y
      DO 2 K=J,N
      KJ=J1+(K*(K-1))/2
      T=A(KJ)*Y
      DO 1 L=K,N
      JJ=(L*(L-1))/2
      LK=K+JJ
      LJ=J1+JJ
    1 A(LK)=A(LK)-T*A(LJ)
      X(K)=X(K)-A(KJ)*X(J1)
    2 A(KJ)=T
      IF(A(KJ+1).LT.ERR)GO TO 5
      X(N)=X(N)/A(LK)
      KJ=N+1
      DO 3 J=2,N
      KL=KJ
      JJ=KL-J
      J1=J-1
      DO 3 K=1,J1
      KL=KL-1
      JD=JJ+(KL*(KL-1))/2
    3 X(JJ)=X(JJ)-X(KL)*A(JD)
      RETURN
    4 X(1)=X(1)/A(1)
      RETURN
    5 IND=1
      RETURN
      END
      SUBROUTINE HFWLIN(N,NT ,M,LV,A,E,B,D1,D2,S1,S2,S3)
C     FOR A GIVEN SYMMETRIC MATRIX A THIS SUBROUTINE USES HOUSEHOLDER'S
C     METHOD TO REDUCE THE MATRIX TO CODIAGONAL FORM, THE QR ALGORITHM
C     TO COMPUTE ALL EIGENVALUES AND WILKINSON'S METHOD TO CALCULATE
C     EIGENVECTORS CORRESPONDING TO A SPECIFIED NUMBER OF THE LARGEST
C     OR SMALLEST EIGENVALUES.
C ***** N =      THE ORDER OF THE INPUT MATRIX
C***** NT =     N*(N+1)/2
C ***** M =      1(-1) MEANS THE EIGENVALUES ARE TO BE IN DESCENDING(ASCENDING)
C *****          ORDER
C ***** LV =      THE NUMBER OF EIGENVECTORS WANTED
C ***** A =      THE GIVEN MATRIX,STORED AS A VECTOR,READING ROW-WISE AND NOT
C *****          INCLUDING THE UPPER TRIANGULAR PART.SHOULD BE DIMENSIONED IN THE
C *****          CALLING PROGRAM BY AT LEAST NT.A WILL BE DESTROYED UPON RETURN
C *****          TO THE CALLING PROGRAM
C ***** E =      THE VECTOR OF EIGENVALUES,SHOULD BE DIMENSIONED IN THE CALLING
C *****          PROGRAM BY AT LEAST N.
C ***** B =      THE MATRIX OF EIGENVECTORS,STORED AS A VECTOR,READING ROW-WISE.
C *****          SHOULD BE DIMENSIONED IN THE CALLING PROGRAM BY AT LEAST N*LV.
C *****          THE EIGENVECTORS ARE NORMALIZED SO THAT B'B=I .
C ***** D1,D2,S1,S2,S3   ARE USED INTERNALLY AND SHOULD BE DIMENSIONED IN THE
C *****          CALLING PROGRAM BY AT LEAST N.
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),E(1),B(1),D1(1),D2(1),S1(1),S2(1),S3(1)
      IF(N.EQ.1)GO TO 250
      VX=1.D+25
      IF(M.GT.0) VX=-VX
      CALL TRIDI (   N,A,D1,D2,S1,S2)
      URM=0.DO
```

```
      DO 210 I=1,N
      S1(I)=D1(I)
      S2(I)=D2(I)**2
  210 ORM=DMAX1(ORM,DABS(S1(I))+S2(I))
      ORM=ORM+1.D0
      CALL QRB (    N,E,S1,S2,ORM,1.D-24)
      DO 216 I=1,N
      VW=VX
      DO 214 J=I,N
      IF(E(J).LE.VW.AND.M.GT.0.OR.E(J).GE.VW.AND.M.LT.0) GO TO 214
      VW=E(J)
      L=J
  214 CONTINUE
      E(L)=E(I)
  216 E(I)=VW
      IF(LV.EQ.0) RETURN
      DO 240 I=1,LV
      EV=E(I)
      CALL EIGVEC (N,NT,A,D1,D2,EV,S1,S2,S3)
      L=I
      DO 240 J=1,N
      B(L)=S1(J)
  240 L=L+LV
      RETURN
  250 E(1)=A(1)
      B(1)=1.D0
      RETURN
      END
      SUBROUTINE QRB (    N,G,A,BQ,ORM,EPS)
C     COMP.J.,1963,6,99-101.
C     FRANCIS, J.G.F. THE QR TRANSFORMATION-PART II.
C     MATRIX BY THE QR METHOD. COMM. ACM,1965,8,217-218.
C     BUSINGER, P.A. ALGORITHM 253-EIGENVALUES OF A REAL SYMMETRIC
C     N - ORDER OF MATRIX
C     G - VECTOR OF EIGENVALUES
C     A - PRINCIPAL DIAGONAL
C     BQ - SQUARED SUBDIAGONAL
C     ORM - MATRIX NORM /COMPUTED IN TRIDI/
C     EPS - RELATIVE MACHINE PRECISION
      EPSQ=EPS*ORM
      DIMENSION A(1),BQ(1),G(1)
      IMPLICIT REAL*8(A-H,O-Z)
      UM=0.D0
      M=N
  200 IF(M.EQ.0) RETURN
      M1=M-1
      I=M1
      K=M1
      BQ(1)=0.D0
      IF(BQ(K+1).GT.EPSQ) GO TO 210
      G(M)=A(M)
      UM=0.D0
      M=K
      GO TO 200
  210 I=I-1
      IF(BQ(I+1).LE.EPSQ) GO TO 211
      K=I
      GO TO 210
  211 IF(K.NE.M1) GO TO 220
```

```
C       TREAT 2X2 BLOCK SEPARATELY
        UM=A(M1)*A(M)-BQ(M1+1)
        SQ1=A(M1)+A(M)
        VW=(A(M1)-A(M))**2+4.D0*BQ(M1+1)
        SQ2=DSQRT(VW)
        VW=SQ1+SQ2
        IF(SQ1.LT.0.D0)VW=SQ1-SQ2
        AMBDA=.5D0*VW
        G(M1)=AMBDA
        G(M)=UM/AMBDA
        UM=0.D0
        M=M-2
        GO TO 200
  220   AMBDA=0.D0
        VA=DABS(A(M)-UM)
        VB=.5D0*DABS(A(M))
        IF(VA.LT.VB) AMBDA=A(M)+.5D0*DSQRT(BQ(M1+1))
        UM=A(M)
        SQ1=0.D0
        SQ2=0.D0
        U=0.D0
        DO 240 I=K,M1
C       SHORTCUT SINGLE QR ITERATION
        GAMMA=A(I)-AMBDA-U
        IF(SQ1.EQ.1.D0)GO TO 221
        PQ=GAMMA**2/(1.D0-SQ1)
        GO TO 222
  221   PQ=(1.D0-SQ2)*BQ(I)
  222   T=PQ+BQ(I+1)
        BQ(I)=SQ1*T
        SQ2=SQ1
        SQ1=BQ(I+1)/T
        U=SQ1*(GAMMA+A(I+1)-AMBDA)
  240   A(I)=GAMMA+U+AMBDA
        GAMMA=A(M)-AMBDA-U
        IF(SQ1.EQ.1.D0) GO TO 241
        VW=GAMMA**2/(1.D0-SQ1)
        GO TO 242
  241   VW=(1.D0-SQ2)*BQ(M1+1)
  242   BQ(M1+1)=SQ1*VW
        A(M)=GAMMA+AMBDA
        GO TO 200
        END
        SUBROUTINE EIGVEC(LP,LPT,R,A,B,E,V,P,Q)
C       WILKINSON, J.H. THE CALCULATION OF EIGENVECTORS OF CODIAGONAL
C       MATRICES. COMP.J.,1958,1,90-96.
C       MATULA, D.W. SHARE PROGRAM SUBMITTAL, 1962 F2 BCHOW.
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION R(1),A(1),B(1),V(1),P(1),Q(1)                    BCHOW186
C  SET UP SIMULTANEOUS EQUATIONS FOR EIGEN VECTOR WITH EIGEN VALUE E BCHOW187
                                                                   BCHOW188
        X=A(1)-E                                                   BCHOW189
        Y=B(2)                                                     BCHOW190
        LP1=LP-1                                                   BCHOW191
        DO 10 I=1,LP1
        IF(DABS(X)-DABS(B(I+1))) 4,6,8                             BCHOW193
    4   P(I)=B(I+1)                                                BCHOW194
        Q(I)=A(I+1)-E                                              BCHOW195
        V(I)=B(I+2)                                                BCHOW196
        Z=-X/P(I)
```

```
                                                                      BCHOW197
      X=Z*Q(I)+Y                                                      BCHOW198
      IF(LP1-I)5,10,5                                                 BCHOW199
    5 Y=Z*V(I)                                                        BCHOW200
      GO TO 10                                                        BCHOW201
    6 IF(X)8,7,8
    7 X=1.0D-10                                                       BCHOW203
    8 P(I)=X                                                          BCHOW204
      Q(I)=Y
      V(I)=0.D0                                                       BCHOW206
      X=A(I+1)-(B(I+1)/X*Y+E)                                         BCHOW207
      Y=B(I+2)                                                        BCHOW208
   10 CONTINUE                                                        BCHOW209
C  SOLVE SIMULTANEOUS EQUATIONS FOR EIGEN VECTOR OF TRI-DIAGONAL MATRIX BCHOW210
   20 IF(X) 21,25,21
   21 V(LP)=1.D0/X                                                    BCHOW212
   22 I=LP1
      V(I)=(1.D0-Q(I)*V(LP))/P(I)                                     BCHOW214
      X=V(LP)**2+V(I)**2                                              BCHOW215
   25 I=I-1                                                           BCHOW216
      IF(I) 26,30,26
   26 V(I)=(1.D0-(Q(I)*V(I+1)+V(I)*V(I+2)))/P(I)                      BCHOW218
      X=X+V(I)**2                                                     BCHOW219
      GO TO 25
   28 V(LP)=1.0D10                                                    BCHOW221
      GO TO 22
   30 X=DSQRT(X)                                                      BCHOW223
      DO 31 I=1,LP                                                    BCHOW224
   31 V(I)=V(I)/X                                                     BCHOW225
C  TRANSFORM EIGEN VECTOR TO SOLUTION OF ORIGINAL MATRIX
      IF(LP.EQ.2)RETURN                                               BCHOW227
      K=LP
      J=LPT-1
      DO 44 N=3,LP
      J=J-K
      K=K-1
      L=J
      Y=0.D0                                                          BCHOW232
      DO 35 I=K,LP
      Y=Y+V(I)*R(L)
   35 L=L+I
      L=J                                                             BCHOW235
      DO 40 I=K,LP
      V(I)=V(I)-Y*R(L)
   40 L=L+I
   44 CONTINUE
      RETURN                                                          BCHOW240
       END
      SUBROUTINE TRIDI(   LP,P,A,B,W,Q)
C     WILKINSON,J.H. HOUSEHOLDER'S METHOD FOR THE SOLUTION OF THE
C     ALGEBRAIC EIGENPROBLEM. COMP.J.,1960,3,23-27.
C     MATULA, D.W. SHARE PROGRAM SUBMITTAL, 1962 F2 BCHOW.
C     MATULA, D.W.  SHARE PROGRAM SUBMITTAL, 1962 F2 BCHOW.           BCHOW033
C  TRI-DIAGONALIZATION SUBROUTINE      DWM       1517-UB
C ***** LP =     ORDER OF THE INPUT MATRIX
C ***** R =      INPUT MATRIX,RETURNS WITH MODIFIED W MATRICES
C ***** A =      NEW DIAGONAL
C ***** B =      NEW FIRST OFF DIAGONAL
C ***** W,Q =    INTERNAL ARRAYS,MUST BE DIMENSIONED IN THE CALLING PROGRAM
C *****          BY AT LEAST LP
      IMPLICIT REAL*8(A-H,O-Z)
```

-A17 39 0A

```
      DIMENSION R(1),A(1),B(1),Q(1),W(1)                                BCHOW039
      LP1=LP-1                                                          BCHOW040
      B(1)=0.D0
      IF(LP-2)99,65,15
   15 KL=0
      DO 51 K=2,LP1
      KL=KL+K
      KJ=K+1
      K1=K-1
C  CALCULATE AND STORE MODIFIED COLUMN MATRIX W                         BCHOW056
      SUM=0.D0
      L=KL
      DO 20 J=K,LP
      SUM=SUM+R(L)**2
      L=L+J
   20 CONTINUE
      S=DSQRT(SUM)
      B(K)=DSIGN(S,-R(KL))
      IF(SUM.LE.1.D-14) GO TO  51
      S=1.D0/S
      W(K)=DSQRT(DABS(R(KL))*S+1.D0)
      X=DSIGN(S/W(K),R(KL))
      R(KL)=W(K)                                                        BCHOW065
      JJ=KL+K
      DO 30 I=KJ,LP                                                     BCHOW066
      W(I)=X*R(JJ)                                                      BCHOW068
      R(JJ)=W(I)
      JJ=JJ+I
C  CALCULATE NEW R MATRIX WITH ROW K-1 NOW HAVING ZEROS OFF 2ND DIAGONALBCHOW070
   30 CONTINUE
      L=KL
      DO 35 J=K,LP                                                      BCHOW071
      JJ=J+1                                                            BCHOW072
      Q(J)=0.D0
      DO 33 I=K,J                                                       BCHOW075
      L=L+1
      Q(J)=Q(J)+R(L)*W(I)
   33 CONTINUE
      L1=L
      L=L+K1
      IF(JJ-LP)34,34,36                                                 BCHOW078
   34 DO 35 I=JJ,LP                                                     BCHOW079
      L1=L1+I-1
      Q(J)=Q(J)+R(L1)*W(I)
   35 CONTINUE
   36 X=0.D0
      DO 40 J=K,LP                                                      BCHOW083
   40 X=X+W(J)*Q(J)                                                     BCHOW084
      X=X*.5D0
      DO 45 I=K,LP                                                      BCHOW086
   45 Q(I)=X*W(I)-Q(I)                                                  BCHOW087
      LL=KL-K1
      DO 50 I=K,LP                                                      BCHOW090
      LL=LL+I
      L=LL
      DO 50 J=I,LP                                                      BCHOW092
      R(L)=R(L)+Q(I)*W(J)+Q(J)*W(I)
      L=L+J
   50 CONTINUE
```

```
   51 CONTINUE
C  SCRT OUTPUT                                                        BCHOW095
   65 L=0
      DO 60 I=1,LP                                                   BCHOW097
      L=L+I
      A(I)=R(L)                                                      BCHOW099
   60 CONTINUE
      LPP=L-1
      B(LP)=R(LPP)
   99 RETURN                                                         BCHOW104
      END
      SUBROUTINE PMSL(N,K,A,FMT,TEXT,LC,LT,IND)
C ***** PRINT MATRIX STORED LINEARLY
C         N,K    ORDER OF MATRIX,I.E. A(NXK)
C         A      MATRIX TO BE PRINTED
C         FMT    VARIABLE FORMAT WITH WHICH A IS PRINTED,SPECIFIED IN THE CALLING
C                PROGRAM THROUGH DATA CARD OR THE LIKE
C         TEXT   HOLLERITH TITLE OF MATRIX,NUMBER OF CHARACTERS IN THIS TITLE
C                SHOULD BE A MULTIPLE OF 4
C         LC     CARRIAGE CONTROL DIGIT (I.E. LC=1 IMPLIES NEW PAGE,LC=0 IMPLIES
C                DOUBLE SPACE  ETC)
C         LT     NUMBER OF WORDS IN TEXT,I.E. NUMBER OF CHARACTERS IN TEXT
C                DIVIDED BY 4,NOT TO BE EXCEEDED BY 20  (SEE DIMENSION)
C         IND    =0 IMPLIES PRINT FULL MATRIX
C                OTHERWISE PRINT SYMMETRIC MATRIX,I.E. ONLY LOWER TRIANGULAR PART
C ***********************************
      IMPLICIT REAL*8(A-H,P-Z)
      REAL TEXT,FMT
      DIMENSION A(1),TEXT(20),FMT(1)
      LO=1
      LL=1
    1 WRITE(6,11)LC,(TEXT(I),I=1,LT)
      L=MINO(LO+9,K)
      WRITE(6,12)(I,I=LO,L)
      IF(IND.EQ.0)GO TO 2
      LL=LO
    2 DO 4 I=LL,N
      IF(IND.EQ.0)GO TO 3
      LCX=(I*(I-1))/2
      LOW=LCX+LO
      LR=LCX+MINO(I,L)
      GO TO 4
    3 LCX=(I-1)*K
      LOW=LCX+LO
      LR=LCX+L
    4 WRITE(6,FMT)I,(A(J),J=LOW,LR)
      IF(L.EQ.K)RETURN
      LO=LO+10
      LC=0
      GO TO 1
   11 FORMAT(I1,10X,20A4)
   12 FORMAT(1HO,10X,10I11)
```

F21P4000-I.N. 348005



A1

## Appendix B

### An Example of Input Data

We shall illustrate how input data should be set up by means of four sets of data. In all four cases Harman's correlation matrix of twenty-four psychological tests for 145 children is the input matrix. In the first set of data all twenty-four variables are to be analyzed with 4 and 5 common factors using the ML method of estimation. Intermediate output is requested.

The second set of data is as the first except that the ULS method of estimation is used and no intermediate output is to be printed.

The third set of data selects the first 13 variables from the input matrix to be analyzed with 4 common factors and using the ML method of estimation. No intermediate output is to be printed.

The last set of data analyzes the matrix of the third data set with the Heywood variable (the 11th variable) removed. Thus 12 variables are selected from the first thirteen and are analyzed with 3 and 4 common factors using the ML method of estimation. The 12 variables selected are also rearranged so that the variables appear in the following order:

13, 10, 12, 5, 4, 9, 1, 8, 7, 2, 6, 3

i.e., the 13th variable is now the first, the 10th variable is now the second, etc.

The next two pages show card by card how the data should be punched. One line corresponds to one card. For all sets of data MAXIT has been set to 30 and the scratch unit is 4.

The results obtained from these data follow on subsequent pages.

```
HARMAN'S 24 PSYCHOLOGICAL TESTS                ML
  145   24    4    5   30     FF          3310       .0005          .1    4
(16F5.0)
   1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
 .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
 .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
 .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
 .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
 .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
 .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
 .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
 .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
 .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
 .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
 .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
 .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
 .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
 .232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
 .160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
 .335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
 .509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
 .358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
HARMAN'S 24 PSYCHOLOGICAL TESTS               ULS
  145   24    4    5   30     FF          3100       .0005          .1    4
(16F5.0)
   1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
 .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
 .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
 .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
 .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
 .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
 .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
 .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
 .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
 .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
 .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
 .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
 .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
 .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
 .232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
 .160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
 .335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
 .509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
 .358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
HARMAN'S 13 PSYCHOLOGICAL TESTS                ML
  145   24    4    4   30     TF          3300       .0005          .1    4
(16F5.0)
   1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
 .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
 .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
 .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
 .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
 .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
 .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
 .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
 .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
 .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
 .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
 .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
 .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
 .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
```

```
   .232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
   .160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
   .335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
   .509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
   .358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
     13
      1    2    3    4    5    6    7    8    9   10   11   12   13
   HARMAN'S 12 PSYCHOLOGICAL TESTS          ML
     145   24    3    4   30     TF        3300      .0005        .1    4
   (16F5.0)
      1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
    .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
    .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
    .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
    .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
    .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
    .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
    .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
    .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
    .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
    .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
    .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
    .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
    .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
   .232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
   .160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
   .335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
   .509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
   .358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
     12
     13   10   12    5    4    9    1    8    7    2    6    3
   STOP
                93
```

-B3-44

F21P4000.1.N 348005

C-43506-8

HARMAN'S 24 PSYCHOLOGICAL TESTS ML

N= 145

P= 24

KL= 4

KU= 5

MAXIT= 30

LOGICAL VARIABLES= FF

INTEGER VARIABLES= 3310

EPS= 0.0005000

EPSE= 0.1000000

LOGICAL SCRATCH TAPE (DISK) NUMBER= 4



FEI P4000-I.N. 34808

MATRIX TO BE ANALYZED

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | 0.318 | 1.000 | | | | | | | | |
| 3 | 0.403 | 0.317 | 1.000 | | | | | | | |
| 4 | 0.458 | 0.230 | 0.305 | 1.000 | | | | | | |
| 5 | 0.321 | 0.285 | 0.247 | 0.227 | 1.000 | | | | | |
| 6 | 0.335 | 0.234 | 0.268 | 0.327 | 0.622 | 1.000 | | | | |
| 7 | 0.304 | 0.157 | 0.223 | 0.335 | 0.656 | 0.722 | 1.000 | | | |
| 8 | 0.332 | 0.157 | 0.382 | 0.391 | 0.578 | 0.527 | 0.619 | 1.000 | | |
| 9 | 0.326 | 0.195 | 0.184 | 0.325 | 0.723 | 0.714 | 0.685 | 0.532 | 1.000 | |
| 10 | 0.116 | 0.057 | -0.075 | 0.099 | 0.311 | 0.203 | 0.246 | 0.285 | 0.170 | 1.000 |
| 11 | 0.308 | 0.150 | 0.091 | 0.110 | 0.344 | 0.353 | 0.232 | 0.300 | 0.280 | 0.484 |
| 12 | 0.314 | 0.145 | 0.140 | 0.160 | 0.215 | 0.095 | 0.181 | 0.271 | 0.113 | 0.585 |
| 13 | 0.489 | 0.239 | 0.321 | 0.327 | 0.344 | 0.309 | 0.345 | 0.395 | 0.280 | 0.408 |
| 14 | 0.125 | 0.103 | 0.177 | 0.066 | 0.280 | 0.292 | 0.236 | 0.252 | 0.260 | 0.172 |
| 15 | 0.238 | 0.131 | 0.065 | 0.127 | 0.229 | 0.251 | 0.172 | 0.175 | 0.248 | 0.154 |
| 16 | 0.414 | 0.272 | 0.263 | 0.322 | 0.187 | 0.291 | 0.180 | 0.296 | 0.242 | 0.124 |
| 17 | 0.176 | 0.005 | 0.177 | 0.187 | 0.208 | 0.273 | 0.228 | 0.255 | 0.274 | 0.289 |
| 18 | 0.368 | 0.255 | 0.211 | 0.251 | 0.263 | 0.167 | 0.159 | 0.250 | 0.208 | 0.317 |
| 19 | 0.270 | 0.112 | 0.312 | 0.137 | 0.190 | 0.251 | 0.226 | 0.274 | 0.274 | 0.190 |
| 20 | 0.365 | 0.292 | 0.297 | 0.339 | 0.398 | 0.435 | 0.451 | 0.427 | 0.446 | 0.173 |
| 21 | 0.369 | 0.306 | 0.165 | 0.349 | 0.318 | 0.263 | 0.314 | 0.362 | 0.266 | 0.405 |
| 22 | 0.413 | 0.232 | 0.250 | 0.380 | 0.441 | 0.386 | 0.396 | 0.357 | 0.483 | 0.160 |
| 23 | 0.474 | 0.348 | 0.383 | 0.335 | 0.435 | 0.431 | 0.405 | 0.501 | 0.504 | 0.262 |
| 24 | 0.282 | 0.211 | 0.203 | 0.248 | 0.420 | 0.433 | 0.437 | 0.388 | 0.424 | 0.531 |

MATRIX TO BE ANALYZED

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 1.000 | | | | | | | | | |
| 12 | 0.428 | 1.000 | | | | | | | | |
| 13 | 0.535 | 0.512 | 1.000 | | | | | | | |
| 14 | 0.350 | 0.131 | 0.195 | 1.000 | | | | | | |
| 15 | 0.240 | 0.173 | 0.139 | 0.370 | 1.000 | | | | | |
| 16 | 0.314 | 0.119 | 0.281 | 0.412 | 0.325 | 1.000 | | | | |
| 17 | 0.362 | 0.278 | 0.194 | 0.341 | 0.345 | 0.324 | 1.000 | | | |
| 18 | 0.350 | 0.349 | 0.323 | 0.201 | 0.334 | 0.344 | 0.448 | 1.000 | | |
| 19 | 0.290 | 0.110 | 0.263 | 0.206 | 0.192 | 0.258 | 0.324 | 0.358 | 1.000 | |
| 20 | 0.202 | 0.246 | 0.241 | 0.302 | 0.272 | 0.388 | 0.262 | 0.301 | 0.167 | 1.000 |
| 21 | 0.399 | 0.355 | 0.425 | 0.183 | 0.232 | 0.348 | 0.173 | 0.357 | 0.331 | 0.413 |
| 22 | 0.304 | 0.193 | 0.279 | 0.243 | 0.246 | 0.283 | 0.273 | 0.317 | 0.342 | 0.463 |
| 23 | 0.251 | 0.350 | 0.382 | 0.242 | 0.256 | 0.360 | 0.287 | 0.272 | 0.303 | 0.509 |
| 24 | 0.412 | 0.414 | 0.358 | 0.304 | 0.165 | 0.262 | 0.326 | 0.405 | 0.374 | 0.366 |

MATRIX TO BE ANALYZED

| | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| 21 | 1.000 | | | |
| 22 | 0.374 | 1.000 | | |
| 23 | 0.451 | 0.503 | 1.000 | |
| 24 | 0.448 | 0.375 | 0.434 | 1.000 |

INTERMEDIATE RESULTS

ITER= 1 F= 0.1973708D 01
PSI= 0.6695372D 00 0.8012888D 00 0.7161767D 00 0.7359262D 00 0.5478115D 00 0.5445127D 00 0.5381649D 00
0.5324443D 00 0.5126799D 00 0.6212547D 00 0.6486282D 00 0.6516243D 00 0.6498442D 00 0.7668719D 00
0.8050684D 00 0.7237529D 00 0.7339121D 00 0.7148514D 00 0.7616699D 00 0.7011788D 00 0.6948178D 00
0.7104082D 00 0.6342621D 00 0.6588050D 00

ITER= 2 F= 0.1717499D 01
PSI= 0.6773125D 00 0.8890447D 00 0.8220106D 00 0.8111837D 00 0.5987694D 00 0.5649063D 00 0.5291028D 00
0.7012642D 00 0.5007284D 00 0.5361959D 00 0.7385593D 00 0.6564233D 00 0.6996297D 00 0.8136112D 00
0.8378412D 00 0.7436853D 00 0.7692774D 00 0.7725429D 00 0.8869384D 00 0.7711841D 00 0.7700830D 00
0.7789265D 00 0.7067444D 00 0.7231099D 00

ITER= 3 F= 0.1711112D 01
PSI= 0.6637980D 00 0.8824492D 00 0.8049010D 00 0.8056633D 00 0.5922127D 00 0.5573563D 00 0.5327706D 00
0.6965713D 00 0.5075525D 00 0.5033638D 00 0.7390441D 00 0.6568816D 00 0.6986178D 00 0.8052307D 00
0.8341358D 00 0.7394329D 00 0.7724677D 00 0.7692933D 00 0.8729829D 00 0.7681030D 00 0.7638858D 00
0.7743646D 00 0.7045731D 00 0.7096049D 00

ITER= 4 F= 0.1710842D 01
PSI= 0.6624246D 00 0.8831198D 00 0.8030295D 00 0.8065156D 00 0.5933544D 00 0.5582383D 00 0.5314280D 00
0.6957135D 00 0.5064482D 00 0.4932306D 00 0.7412843D 00 0.6584495D 00 0.6998659D 00 0.8037167D 00
0.8344472D 00 0.7402953D 00 0.7732623D 00 0.7698190D 00 0.8726947D 00 0.7689338D 00 0.7637106D 00
0.7752680D 00 0.7050339D 00 0.7076906D 00

ITER= 5 F= 0.1710821D 01
PSI= 0.6621638D 00 0.8832289D 00 0.8021929D 00 0.8069802D 00 0.5933004D 00 0.5581273D 00 0.5316019D 00
0.6966785D 00 0.5065494D 00 0.4896464D 00 0.7422758D 00 0.6595946D 00 0.7005154D 00 0.8037265D 00
0.8342561D 00 0.7410087D 00 0.7734042D 00 0.7698349D 00 0.8726417D 00 0.7691675D 00 0.7634811D 00
0.7752598D 00 0.7051682D 00 0.7069368D 00

ITER= 6 F= 0.1710821D 01
PSI= 0.6621666D 00 0.8832292D 00 0.8021943D 00 0.8069813D 00 0.5933005D 00 0.5581276D 00 0.5316027D 00
0.6966785D 00 0.5065487D 00 0.4895842D 00 0.7422800D 00 0.6596047D 00 0.7005202D 00 0.8037259D 00
0.8342656D 00 0.7410119D 00 0.7734036D 00 0.7698353D 00 0.8726416D 00 0.7691681D 00 0.7634810D 00
0.7752599D 00 0.7051682D 00 0.7069409D 00

MAXIMUM LIKELIHOOD SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.553 | 0.044 | 0.454 | -0.218 |
| 2 | -0.344 | -0.010 | 0.289 | -0.134 |
| 3 | -0.377 | -0.111 | 0.421 | -0.158 |
| 4 | -0.465 | -0.071 | 0.298 | -0.198 |
| 5 | -0.741 | -0.225 | -0.217 | -0.038 |
| 6 | -0.737 | -0.347 | -0.145 | 0.060 |
| 7 | -0.738 | -0.325 | -0.242 | -0.096 |
| 8 | -0.696 | -0.121 | -0.033 | -0.120 |
| 9 | -0.749 | -0.391 | -0.160 | 0.060 |
| 10 | -0.486 | 0.617 | -0.378 | -0.014 |
| 11 | -0.540 | 0.370 | -0.039 | 0.138 |
| 12 | -0.447 | 0.572 | -0.040 | -0.191 |
| 13 | -0.579 | 0.307 | 0.117 | -0.258 |
| 14 | -0.404 | 0.045 | 0.082 | 0.427 |
| 15 | -0.365 | 0.071 | 0.162 | 0.374 |
| 16 | -0.452 | 0.072 | 0.419 | 0.256 |
| 17 | -0.438 | 0.190 | 0.081 | 0.409 |
| 18 | -0.464 | 0.315 | 0.245 | 0.181 |
| 19 | -0.415 | 0.093 | 0.174 | 0.164 |
| 20 | -0.602 | -0.091 | 0.191 | 0.037 |
| 21 | -0.561 | 0.271 | 0.146 | -0.090 |
| 22 | -0.595 | -0.081 | 0.193 | 0.038 |
| 23 | -0.669 | -0.001 | 0.215 | -0.090 |
| 24 | -0.654 | 0.237 | -0.112 | 0.055 |

UNIQUE VARIANCES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.438 | 0.780 | 0.644 | 0.651 | 0.352 | 0.312 | 0.283 | 0.485 | 0.257 | 0.240 |
| 0.551 | 0.435 | 0.491 | 0.646 | 0.696 | 0.549 | 0.598 | 0.593 | 0.762 | 0.592 |
| 0.583 | 0.601 | 0.497 | 0.500 | | | | | | |

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.160 | 0.187 | 0.689 | 0.160 |
| 2 | 0.117 | 0.083 | 0.436 | 0.096 |
| 3 | 0.137 | -0.019 | 0.570 | 0.110 |
| 4 | 0.233 | 0.099 | 0.527 | 0.080 |
| 5 | 0.739 | 0.213 | 0.185 | 0.150 |
| 6 | 0.767 | 0.066 | 0.205 | 0.233 |
| 7 | 0.806 | 0.153 | 0.197 | 0.075 |
| 8 | 0.569 | 0.242 | 0.338 | 0.132 |
| 9 | 0.806 | 0.040 | 0.201 | 0.227 |
| 10 | 0.168 | 0.831 | -0.118 | 0.167 |
| 11 | 0.180 | 0.512 | 0.120 | 0.374 |
| 12 | 0.019 | 0.716 | 0.210 | 0.088 |
| 13 | 0.198 | 0.525 | 0.438 | 0.082 |
| 14 | 0.197 | 0.081 | 0.050 | 0.553 |
| 15 | 0.122 | 0.074 | 0.116 | 0.520 |
| 16 | 0.069 | 0.062 | 0.408 | 0.525 |
| 17 | 0.142 | 0.219 | 0.062 | 0.574 |
| 18 | 0.026 | 0.336 | 0.293 | 0.456 |
| 19 | 0.148 | 0.161 | 0.239 | 0.365 |
| 20 | 0.378 | 0.118 | 0.402 | 0.301 |
| 21 | 0.175 | 0.438 | 0.381 | 0.223 |
| 22 | 0.366 | 0.122 | 0.399 | 0.301 |
| 23 | 0.369 | 0.244 | 0.500 | 0.239 |
| 24 | 0.370 | 0.496 | 0.157 | 0.304 |

-B7-

P21P4000-I.N. 348008

C1 43506 8

CHISQUARE WITH 185 DEGREES OF FREEDOM IS 226.6835

PROBABILITY LEVEL IS 0.022

TUCKER'S RELIABILITY COEFFICIENT= 0.952

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.2414D 01 | - - |
| 2 | 0.2177D 01 | 0.2365D 00 |
| 3 | 0.1832D 01 | 0.3451D 00 |
| 4 | 0.1706D 01 | 0.1324D 00 |
| 5 | 0.1551D 01 | 0.1486D 00 |
| 6 | 0.1467D 01 | 0.8450D-01 |
| 7 | 0.1385D 01 | 0.8138D-01 |
| 8 | 0.1292D 01 | 0.9345D-01 |
| 9 | 0.1161D 01 | 0.1309D 00 |
| 10 | 0.1091D 01 | 0.6969D-01 |
| 11 | 0.1054D 01 | 0.3691D-01 |
| 12 | 0.9810D 00 | 0.7336D-01 |
| 13 | 0.9003D 00 | 0.8066D-01 |
| 14 | 0.8404D 00 | 0.5988D-01 |
| 15 | 0.8033D 00 | 0.3713D-01 |
| 16 | 0.6883D 00 | 0.1150D 00 |
| 17 | 0.6751D 00 | 0.1325D-01 |
| 18 | 0.6417D 00 | 0.3342D-01 |
| 19 | 0.6224D 00 | 0.1932D-01 |
| 20 | 0.5620D 00 | 0.6035D-01 |
| 21 | 0.3861D 00 | 0.1759D 00 |
| 22 | 0.2546D 00 | 0.1315D 00 |
| 23 | 0.1706D 00 | 0.8401D-01 |
| 24 | 0.5347D-01 | 0.1171D 00 |

IC-B9-50

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | -0.055 | 1.000 | | | | | | | | |
| 3 | -0.049 | 0.061 | 1.000 | | | | | | | |
| 4 | 0.067 | -0.061 | -0.053 | 1.000 | | | | | | |
| 5 | 0.028 | 0.163 | 0.059 | -0.159 | 1.000 | | | | | |
| 6 | 0.058 | 0.055 | 0.050 | 0.033 | -0.095 | 1.000 | | | | |
| 7 | -0.004 | -0.092 | -0.011 | 0.051 | -0.063 | 0.122 | 1.000 | | | |
| 8 | -0.128 | -0.146 | 0.181 | 0.080 | 0.057 | -0.066 | 0.126 | 1.000 | | |
| 9 | 0.042 | -0.028 | -0.160 | 0.021 | 0.158 | -0.003 | -0.102 | -0.098 | 1.000 | |
| 10 | -0.035 | 0.009 | -0.083 | 0.068 | 0.025 | 0.018 | -0.020 | 0.020 | -0.051 | 1.000 |
| 11 | 0.083 | -0.003 | -0.056 | -0.127 | 0.054 | 0.168 | -0.108 | -0.031 | 0.015 | -0.055 |
| 12 | 0.041 | -0.029 | 0.041 | -0.062 | -0.009 | -0.083 | 0.024 | 0.009 | 0.019 | -0.009 |
| 13 | 0.099 | -0.041 | 0.084 | -0.011 | 0.001 | 0.056 | 0.057 | 0.005 | 0.003 | -0.063 |
| 14 | -0.084 | -0.003 | 0.097 | -0.090 | 0.052 | -0.009 | 0.031 | 0.054 | -0.093 | -0.038 |
| 15 | 0.074 | 0.013 | -0.110 | -0.017 | 0.048 | 0.017 | 0.002 | -0.035 | 0.013 | -0.002 |
| 16 | 0.053 | 0.046 | -0.060 | 0.071 | -0.071 | 0.069 | -0.011 | 0.067 | -0.045 | 0.059 |
| 17 | -0.044 | -0.164 | 0.102 | 0.086 | -0.090 | 0.007 | 0.060 | 0.046 | 0.020 | -0.014 |
| 18 | 0.050 | 0.077 | -0.005 | 0.033 | 0.109 | -0.096 | -0.013 | -0.010 | 0.029 | -0.021 |
| 19 | -0.013 | -0.076 | 0.169 | -0.098 | -0.102 | -0.015 | 0.016 | 0.035 | 0.039 | -0.003 |
| 20 | -0.084 | 0.050 | -0.023 | 0.005 | -0.056 | -0.034 | 0.066 | 0.015 | -0.031 | 0.024 |
| 21 | -0.077 | 0.091 | -0.150 | 0.075 | -0.018 | -0.070 | 0.036 | -0.003 | -0.051 | 0.052 |
| 22 | 0.016 | -0.035 | -0.094 | 0.077 | 0.055 | -0.127 | -0.046 | -0.103 | 0.087 | -0.016 |
| 23 | -0.030 | 0.070 | 0.045 | -0.102 | -0.042 | -0.066 | -0.122 | 0.064 | 0.118 | 0.050 |
| 24 | -0.058 | 0.046 | 0.069 | 0.009 | -0.080 | 0.034 | 0.025 | -0.073 | 0.015 | 0.071 |

RESIDUAL CORRELATIONS

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 1.000 | | | | | | | | | |
| 12 | -0.001 | 1.000 | | | | | | | | |
| 13 | 0.287 | 0.072 | 1.000 | | | | | | | |
| 14 | 0.100 | 0.018 | 0.086 | 1.000 | | | | | | |
| 15 | -0.046 | 0.086 | -0.028 | 0.070 | 1.000 | | | | | |
| 16 | 0.044 | -0.121 | 0.027 | 0.139 | -0.014 | 1.000 | | | | |
| 17 | 0.003 | 0.107 | -0.040 | -0.041 | 0.009 | -0.046 | 1.000 | | | |
| 18 | -0.057 | 0.012 | -0.045 | -0.159 | 0.054 | -0.066 | 0.152 | 1.000 | | |
| 19 | 0.024 | -0.158 | 0.026 | -0.072 | -0.076 | -0.080 | 0.064 | 0.094 | 1.000 | |
| 20 | -0.153 | 0.085 | -0.171 | 0.051 | 0.022 | 0.058 | -0.026 | -0.006 | -0.170 | 1.000 |
| 21 | 0.025 | -0.123 | -0.043 | -0.048 | 0.029 | 0.064 | -0.168 | -0.014 | 0.093 | 0.128 |
| 22 | 0.026 | -0.024 | -0.098 | -0.041 | -0.016 | -0.123 | -0.006 | 0.020 | 0.092 | 0.100 |
| 23 | -0.171 | 0.091 | -0.109 | -0.013 | 0.018 | -0.019 | 0.024 | -0.138 | 0.004 | 0.126 |
| 24 | -0.079 | -0.018 | -0.133 | 0.026 | -0.158 | -0.035 | -0.036 | 0.081 | 0.147 | 0.024 |

RESIDUAL CORRELATIONS

| | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| 21 | 1.000 | | | |
| 22 | 0.063 | 1.000 | | |
| 23 | 0.067 | 0.122 | 1.000 | |
| 24 | 0.071 | 0.045 | 0.051 | 1.000 |

INTERMEDIATE RESULTS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ITER= 1 | F= | 0.16675710 01 | | | | | | |
| | PSI= | 0.66203340 00 | 0.79213090 00 | 0.70799160 00 | 0.72751530 00 | 0.54155050 00 | 0.53928950 00 | 0.53201430 00 |
| | | 0.62511720 00 | 0.50682050 00 | 0.61415440 00 | 0.64121510 00 | 0.54417690 00 | 0.64241720 00 | 0.75810730 00 |
| | | 0.79586730 00 | 0.71548120 00 | 0.72552430 00 | 0.70668140 00 | 0.75296480 00 | 0.69316500 00 | 0.68687680 00 |
| | | 0.70228900 00 | 0.62701310 00 | 0.65127560 00 | | | | |
| ITER= 2 | F= | 0.14281130 01 | | | | | | |
| | PSI= | 0.67179670 00 | 0.89086330 00 | 0.81421320 00 | 0.81602200 00 | 0.60197110 00 | 0.53901430 00 | 0.53052800 00 |
| | | 0.70419990 00 | 0.50158780 00 | 0.50917420 00 | 0.65330260 00 | 0.67222830 00 | 0.55120770 00 | 0.80486130 00 |
| | | 0.84117010 00 | 0.73984120 00 | 0.77464160 00 | 0.77307500 00 | 0.89025380 00 | 0.72121280 00 | 0.74825660 00 |
| | | 0.75913930 00 | 0.66802500 00 | 0.70659610 00 | | | | |
| ITER= 3 | F= | 0.14175440 01 | | | | | | |
| | PSI= | 0.66844390 00 | 0.88234490 00 | 0.80265690 00 | 0.80616180 00 | 0.59548500 00 | 0.53511820 00 | 0.52979190 00 |
| | | 0.69820780 00 | 0.51188830 00 | 0.47459670 00 | 0.62852930 00 | 0.66592190 00 | 0.51433880 00 | 0.79818390 00 |
| | | 0.83800820 00 | 0.73821300 00 | 0.78037010 00 | 0.77119730 00 | 0.87445920 00 | 0.72071700 00 | 0.74944670 00 |
| | | 0.75978960 00 | 0.66352430 00 | 0.69297210 00 | | | | |
| ITER= 4 | F= | 0.14171200 01 | | | | | | |
| | PSI= | 0.67004730 00 | 0.88338700 00 | 0.80014760 00 | 0.80570290 00 | 0.59709080 00 | 0.53687060 00 | 0.52705260 00 |
| | | 0.69718000 00 | 0.51139860 00 | 0.46638490 00 | 0.62296950 00 | 0.66580650 00 | 0.50700880 00 | 0.79826520 00 |
| | | 0.83953290 00 | 0.74037190 00 | 0.78249760 00 | 0.77172010 00 | 0.87407510 00 | 0.72154450 00 | 0.75043360 00 |
| | | 0.76097200 00 | 0.66475600 00 | 0.69140230 00 | | | | |
| ITER= 5 | F= | 0.14170950 01 | | | | | | |
| | PSI= | 0.67082090 00 | 0.88367510 00 | 0.79918120 00 | 0.80543220 00 | 0.59717130 00 | 0.53683490 00 | 0.52643150 00 |
| | | 0.69661850 00 | 0.51191170 00 | 0.46354410 00 | 0.62116290 00 | 0.66629910 00 | 0.50581170 00 | 0.79911390 00 |
| | | 0.83994390 00 | 0.74162180 00 | 0.78333580 00 | 0.77175150 00 | 0.87390330 00 | 0.72177580 00 | 0.75078850 00 |
| | | 0.76131140 00 | 0.66519710 00 | 0.69110120 00 | | | | |
| ITER= 6 | F= | 0.14170950 01 | | | | | | |
| | PSI= | 0.67081940 00 | 0.88367630 00 | 0.79918300 00 | 0.80543190 00 | 0.59717110 00 | 0.53683660 00 | 0.52643030 00 |
| | | 0.69661790 00 | 0.51191000 00 | 0.46349180 00 | 0.62114480 00 | 0.66627680 00 | 0.50582360 00 | 0.79911790 00 |
| | | 0.83994700 00 | 0.74162470 00 | 0.78333890 00 | 0.77175290 00 | 0.87390380 00 | 0.72177660 00 | 0.75079110 00 |
| | | 0.76131080 00 | 0.66519790 00 | 0.69110710 00 | | | | |

F31P4000-I.N. 348008

C1 43508

MAXIMUM LIKELIHOOD SOLUTION FOR 5 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0.560 | 0.020 | 0.462 | -0.147 | 0.020 |
| 2 | 0.344 | -0.034 | 0.288 | -0.044 | 0.119 |
| 3 | 0.376 | -0.131 | 0.433 | -0.120 | 0.027 |
| 4 | 0.462 | -0.099 | 0.298 | -0.118 | 0.160 |
| 5 | 0.727 | -0.253 | -0.219 | -0.059 | -0.015 |
| 6 | 0.723 | -0.379 | -0.162 | 0.001 | -0.141 |
| 7 | 0.722 | -0.354 | -0.240 | -0.136 | 0.017 |
| 8 | 0.689 | -0.153 | -0.030 | -0.105 | 0.066 |
| 9 | 0.726 | -0.424 | -0.171 | 0.025 | -0.029 |
| 10 | 0.512 | 0.595 | -0.389 | 0.039 | 0.128 |
| 11 | 0.574 | 0.380 | -0.036 | 0.074 | -0.366 |
| 12 | 0.476 | 0.546 | -0.016 | -0.119 | 0.131 |
| 13 | 0.626 | 0.346 | 0.191 | -0.385 | -0.217 |
| 14 | 0.403 | 0.013 | 0.051 | 0.369 | -0.245 |
| 15 | 0.359 | 0.026 | 0.123 | 0.373 | -0.104 |
| 16 | 0.454 | 0.032 | 0.386 | 0.288 | -0.105 |
| 17 | 0.437 | 0.139 | 0.039 | 0.402 | -0.113 |
| 18 | 0.471 | 0.265 | 0.218 | 0.253 | 0.031 |
| 19 | 0.417 | 0.063 | 0.155 | 0.177 | -0.057 |
| 20 | 0.592 | -0.152 | 0.172 | 0.153 | 0.228 |
| 21 | 0.573 | 0.234 | 0.144 | 0.020 | 0.177 |
| 22 | 0.587 | -0.126 | 0.176 | 0.122 | 0.119 |
| 23 | 0.668 | -0.054 | 0.213 | 0.025 | 0.251 |
| 24 | 0.658 | 0.186 | -0.134 | 0.132 | 0.138 |

UNIQUE VARIANCES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.450 | 0.781 | 0.639 | 0.649 | 0.357 | 0.288 | 0.277 | 0.485 | 0.262 | 0.215 |
| 0.386 | 0.444 | 0.256 | 0.639 | 0.706 | 0.550 | 0.614 | 0.596 | 0.764 | 0.521 |
| 0.564 | 0.580 | 0.442 | 0.478 | | | | | | |

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0.160 | 0.136 | 0.658 | 0.182 | 0.201 |
| 2 | 0.113 | 0.074 | 0.435 | 0.107 | 0.003 |
| 3 | 0.134 | -0.054 | 0.561 | 0.107 | 0.117 |
| 4 | 0.231 | 0.092 | 0.533 | 0.071 | 0.010 |
| 5 | 0.736 | 0.192 | 0.188 | 0.162 | 0.061 |
| 6 | 0.775 | 0.029 | 0.187 | 0.251 | 0.113 |
| 7 | 0.809 | 0.135 | 0.208 | 0.069 | 0.047 |
| 8 | 0.568 | 0.223 | 0.349 | 0.131 | 0.059 |
| 9 | 0.800 | 0.030 | 0.216 | 0.224 | -0.001 |
| 10 | 0.175 | 0.844 | -0.100 | 0.176 | 0.035 |
| 11 | 0.185 | 0.436 | 0.057 | 0.451 | 0.428 |
| 12 | 0.023 | 0.690 | 0.222 | 0.101 | 0.142 |
| 13 | 0.196 | 0.456 | 0.424 | 0.084 | 0.561 |
| 14 | 0.197 | 0.055 | 0.050 | 0.556 | 0.086 |
| 15 | 0.121 | 0.066 | 0.130 | 0.508 | -0.026 |
| 16 | 0.067 | 0.037 | 0.400 | 0.529 | 0.066 |
| 17 | 0.145 | 0.208 | 0.078 | 0.562 | -0.006 |
| 18 | 0.025 | 0.329 | 0.306 | 0.452 | 0.006 |
| 19 | 0.147 | 0.145 | 0.242 | 0.364 | 0.053 |
| 20 | 0.370 | 0.140 | 0.453 | 0.287 | -0.189 |
| 21 | 0.170 | 0.439 | 0.403 | 0.230 | -0.001 |
| 22 | 0.353 | 0.126 | 0.423 | 0.302 | -0.078 |
| 23 | 0.360 | 0.257 | 0.549 | 0.223 | -0.106 |
| 24 | 0.371 | 0.502 | 0.185 | 0.307 | -0.057 |

CHISQUARE WITH 166 DEGREES OF FREEDOM IS 186.8203

PROBABILITY LEVEL IS 0.129

TUCKER'S RELIABILITY COEFFICIENT= 0.973

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.2318D 01 | - - |
| 2 | 0.2050D 01 | 0.2677D 00 |
| 3 | 0.1748D 01 | 0.3021D 00 |
| 4 | 0.1648D 01 | 0.1005D 00 |
| 5 | 0.1489D 01 | 0.1585D 00 |
| 6 | 0.1434D 01 | 0.5479D-01 |
| 7 | 0.1301D 01 | 0.1332D 00 |
| 8 | 0.1163D 01 | 0.1381D 00 |
| 9 | 0.1089D 01 | 0.7426D-01 |
| 10 | 0.1021D 01 | 0.6779D-01 |
| 11 | 0.9760D 00 | 0.4510D-01 |
| 12 | 0.9261D 00 | 0.4983D-01 |
| 13 | 0.8573D 00 | 0.6883D-01 |
| 14 | 0.8069D 00 | 0.5041D-01 |
| 15 | 0.7718D 00 | 0.3509D-01 |
| 16 | 0.6814D 00 | 0.9038D-01 |
| 17 | 0.6611D 00 | 0.2033D-01 |
| 18 | 0.6325D 00 | 0.2859D-01 |
| 19 | 0.6096D 00 | 0.2286D-01 |
| 20 | 0.4332D 00 | 0.1765D 00 |
| 21 | 0.3434D 00 | 0.8983D-01 |
| 22 | 0.2435D 00 | 0.9981D-01 |
| 23 | 0.1545D 00 | 0.8905D-01 |
| 24 | 0.4958D-01 | 0.1049D 00 |

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | -0.023 | 1.000 | | | | | | | | |
| 3 | -0.044 | 0.070 | 1.000 | | | | | | | |
| 4 | 0.098 | -0.060 | -0.045 | 1.000 | | | | | | |
| 5 | 0.029 | 0.168 | 0.060 | -0.151 | 1.000 | | | | | |
| 6 | 0.042 | 0.075 | 0.047 | 0.062 | -0.113 | 1.000 | | | | |
| 7 | -0.009 | -0.091 | -0.019 | 0.046 | -0.058 | 0.106 | 1.000 | | | |
| 8 | -0.116 | -0.145 | 0.181 | 0.078 | 0.065 | -0.065 | 0.122 | 1.000 | | |
| 9 | 0.032 | -0.034 | -0.164 | 0.016 | 0.169 | -0.012 | -0.098 | -0.095 | 1.000 | |
| 10 | -0.000 | -0.001 | -0.055 | 0.058 | 0.029 | 0.053 | -0.015 | 0.021 | -0.058 | 1.000 |
| 11 | 0.033 | 0.042 | -0.082 | -0.078 | 0.039 | 0.075 | -0.121 | -0.015 | 0.019 | -0.019 |
| 12 | 0.054 | -0.028 | 0.041 | -0.066 | -0.003 | -0.072 | 0.025 | 0.011 | 0.010 | -0.006 |
| 13 | -0.029 | -0.025 | 0.019 | 0.012 | -0.026 | -0.045 | 0.045 | -0.012 | 0.030 | -0.006 |
| 14 | -0.122 | -0.006 | 0.087 | -0.080 | 0.041 | -0.049 | 0.039 | 0.058 | -0.085 | -0.013 |
| 15 | 0.065 | 0.002 | -0.108 | -0.018 | 0.044 | 0.014 | 0.009 | -0.033 | 0.016 | 0.003 |
| 16 | 0.051 | 0.047 | -0.056 | 0.086 | -0.078 | 0.057 | -0.006 | 0.071 | -0.047 | 0.074 |
| 17 | -0.054 | -0.174 | 0.104 | 0.084 | -0.093 | -0.000 | 0.068 | 0.047 | 0.023 | -0.009 |
| 18 | 0.067 | 0.068 | 0.006 | 0.032 | 0.111 | -0.081 | -0.002 | -0.006 | 0.027 | -0.030 |
| 19 | -0.015 | -0.077 | 0.171 | -0.093 | -0.103 | -0.021 | 0.020 | 0.038 | 0.041 | -0.001 |
| 20 | -0.052 | 0.020 | -0.014 | -0.033 | -0.048 | 0.023 | 0.073 | 0.003 | -0.044 | -0.025 |
| 21 | -0.048 | 0.082 | -0.141 | 0.064 | -0.009 | -0.036 | 0.043 | -0.005 | -0.057 | 0.013 |
| 22 | 0.041 | -0.050 | -0.086 | 0.064 | 0.066 | -0.100 | -0.039 | -0.107 | 0.087 | -0.049 |
| 23 | 0.002 | 0.044 | 0.054 | -0.148 | -0.030 | -0.006 | -0.131 | 0.054 | 0.117 | 0.005 |
| 24 | -0.026 | 0.031 | 0.090 | -0.007 | -0.074 | 0.069 | 0.031 | -0.076 | 0.008 | 0.026 |

RESIDUAL CORRELATIONS

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 1.000 | | | | | | | | | |
| 12 | 0.009 | 1.000 | | | | | | | | |
| 13 | -0.000 | 0.033 | 1.000 | | | | | | | |
| 14 | -0.002 | 0.017 | 0.043 | 1.000 | | | | | | |
| 15 | -0.071 | 0.085 | 0.006 | 0.083 | 1.000 | | | | | |
| 16 | -0.009 | -0.121 | 0.001 | 0.130 | -0.007 | 1.000 | | | | |
| 17 | -0.023 | 0.110 | -0.013 | -0.024 | 0.027 | -0.037 | 1.000 | | | |
| 18 | -0.042 | 0.019 | -0.002 | -0.144 | 0.062 | -0.056 | 0.163 | 1.000 | | |
| 19 | -0.002 | -0.157 | 0.015 | -0.072 | -0.068 | -0.077 | 0.072 | 0.101 | 1.000 | |
| 20 | -0.003 | 0.080 | -0.004 | 0.096 | 0.014 | 0.070 | -0.032 | -0.037 | -0.176 | 1.000 |
| 21 | 0.107 | -0.128 | 0.010 | -0.037 | 0.021 | 0.067 | -0.177 | -0.029 | 0.093 | 0.075 |
| 22 | 0.119 | -0.031 | -0.015 | -0.028 | -0.026 | -0.124 | -0.014 | 0.002 | 0.095 | 0.037 |
| 23 | -0.033 | 0.080 | 0.018 | 0.028 | 0.015 | -0.009 | 0.024 | -0.173 | 0.008 | 0.016 |
| 24 | -0.001 | -0.011 | -0.035 | 0.051 | -0.164 | -0.028 | -0.037 | 0.070 | 0.154 | -0.049 |

RESIDUAL CORRELATIONS

| | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| 21 | 1.000 | | | |
| 22 | 0.031 | 1.000 | | |
| 23 | 0.010 | 0.067 | 1.000 | |
| 24 | 0.037 | 0.006 | -0.010 | 1.000 |



F21P4000-I.M. 348008
C 43508 8

HARMAN'S 24 PSYCHOLOGICAL TESTS ULS

N= 145

P= 24

KL= 4

KU= 5

MAXIT= 30

LOGICAL VARIABLES= FF

INTEGER VARIABLES= 3100

EPS= 0.0005000

EPSE= 0.1000000

LOGICAL SCRATCH TAPE (DISK) NUMBER= 4

MATRIX TO BE ANALYZED

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | 0.318 | 1.000 | | | | | | | | |
| 3 | 0.403 | 0.317 | 1.000 | | | | | | | |
| 4 | 0.468 | 0.230 | 0.305 | 1.000 | | | | | | |
| 5 | 0.321 | 0.285 | 0.247 | 0.227 | 1.000 | | | | | |
| 6 | 0.335 | 0.234 | 0.268 | 0.327 | 0.622 | 1.000 | | | | |
| 7 | 0.304 | 0.157 | 0.223 | 0.335 | 0.656 | 0.722 | 1.000 | | | |
| 8 | 0.332 | 0.157 | 0.382 | 0.391 | 0.578 | 0.527 | 0.619 | 1.000 | | |
| 9 | 0.326 | 0.195 | 0.184 | 0.325 | 0.723 | 0.714 | 0.685 | 0.532 | 1.000 | |
| 10 | 0.116 | 0.057 | -0.075 | 0.099 | 0.311 | 0.203 | 0.246 | 0.285 | 0.170 | 1.000 |
| 11 | 0.308 | 0.150 | 0.091 | 0.110 | 0.344 | 0.353 | 0.232 | 0.300 | 0.280 | 0.484 |
| 12 | 0.314 | 0.145 | 0.140 | 0.160 | 0.215 | 0.095 | 0.181 | 0.271 | 0.113 | 0.585 |
| 13 | 0.489 | 0.239 | 0.321 | 0.327 | 0.344 | 0.309 | 0.345 | 0.395 | 0.280 | 0.408 |
| 14 | 0.125 | 0.103 | 0.177 | 0.066 | 0.280 | 0.292 | 0.236 | 0.252 | 0.260 | 0.172 |
| 15 | 0.238 | 0.131 | 0.065 | 0.127 | 0.229 | 0.251 | 0.172 | 0.175 | 0.248 | 0.154 |
| 16 | 0.414 | 0.272 | 0.263 | 0.322 | 0.187 | 0.291 | 0.180 | 0.296 | 0.242 | 0.124 |
| 17 | 0.176 | 0.005 | 0.177 | 0.187 | 0.208 | 0.273 | 0.228 | 0.255 | 0.274 | 0.289 |
| 18 | 0.368 | 0.255 | 0.211 | 0.251 | 0.263 | 0.167 | 0.159 | 0.250 | 0.208 | 0.317 |
| 19 | 0.270 | 0.112 | 0.312 | 0.137 | 0.190 | 0.251 | 0.226 | 0.274 | 0.274 | 0.190 |
| 20 | 0.365 | 0.292 | 0.297 | 0.339 | 0.398 | 0.435 | 0.451 | 0.427 | 0.446 | 0.173 |
| 21 | 0.369 | 0.306 | 0.165 | 0.349 | 0.318 | 0.263 | 0.314 | 0.362 | 0.266 | 0.405 |
| 22 | 0.413 | 0.232 | 0.250 | 0.380 | 0.441 | 0.386 | 0.396 | 0.357 | 0.483 | 0.160 |
| 23 | 0.474 | 0.348 | 0.383 | 0.335 | 0.435 | 0.431 | 0.405 | 0.501 | 0.504 | 0.262 |
| 24 | 0.282 | 0.211 | 0.203 | 0.248 | 0.420 | 0.433 | 0.437 | 0.388 | 0.424 | 0.531 |

MATRIX TO BE ANALYZED

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 1.000 | | | | | | | | | |
| 12 | 0.428 | 1.000 | | | | | | | | |
| 13 | 0.535 | 0.512 | 1.000 | | | | | | | |
| 14 | 0.350 | 0.131 | 0.195 | 1.000 | | | | | | |
| 15 | 0.240 | 0.173 | 0.139 | 0.370 | 1.000 | | | | | |
| 16 | 0.314 | 0.119 | 0.281 | 0.412 | 0.325 | 1.000 | | | | |
| 17 | 0.362 | 0.278 | 0.194 | 0.341 | 0.345 | 0.324 | 1.000 | | | |
| 18 | 0.350 | 0.349 | 0.323 | 0.201 | 0.334 | 0.344 | 0.448 | 1.000 | | |
| 19 | 0.290 | 0.110 | 0.263 | 0.206 | 0.192 | 0.258 | 0.324 | 0.358 | 1.000 | |
| 20 | 0.202 | 0.246 | 0.241 | 0.302 | 0.272 | 0.388 | 0.262 | 0.301 | 0.167 | 1.000 |
| 21 | 0.399 | 0.355 | 0.425 | 0.183 | 0.232 | 0.348 | 0.173 | 0.357 | 0.331 | 0.413 |
| 22 | 0.304 | 0.193 | 0.279 | 0.243 | 0.246 | 0.283 | 0.273 | 0.317 | 0.342 | 0.463 |
| 23 | 0.251 | 0.350 | 0.382 | 0.242 | 0.256 | 0.360 | 0.287 | 0.272 | 0.303 | 0.509 |
| 24 | 0.412 | 0.414 | 0.358 | 0.304 | 0.165 | 0.262 | 0.326 | 0.405 | 0.374 | 0.366 |

MATRIX TO BE ANALYZED

| | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| 21 | 1.000 | | | |
| 22 | 0.374 | 1.000 | | |
| 23 | 0.451 | 0.503 | 1.000 | |
| 24 | 0.448 | 0.375 | 0.434 | 1.000 |

UNWEIGHTED LEAST SQUARES SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.598 | 0.029 | -0.380 | 0.217 |
| 2 | -0.372 | -0.030 | -0.261 | 0.149 |
| 3 | -0.420 | -0.118 | -0.364 | 0.125 |
| 4 | -0.484 | -0.108 | -0.260 | 0.190 |
| 5 | -0.688 | -0.298 | 0.275 | 0.037 |
| 6 | -0.687 | -0.399 | 0.198 | -0.078 |
| 7 | -0.678 | -0.413 | 0.303 | 0.076 |
| 8 | -0.675 | -0.195 | 0.092 | 0.106 |
| 9 | -0.697 | -0.449 | 0.226 | -0.079 |
| 10 | -0.475 | 0.528 | 0.478 | 0.103 |
| 11 | -0.556 | 0.356 | 0.164 | -0.089 |
| 12 | -0.470 | 0.501 | 0.143 | 0.244 |
| 13 | -0.599 | 0.262 | -0.015 | 0.289 |
| 14 | -0.425 | 0.058 | -0.006 | -0.424 |
| 15 | -0.391 | 0.095 | -0.094 | -0.370 |
| 16 | -0.511 | 0.086 | -0.349 | -0.247 |
| 17 | -0.466 | 0.207 | 0.006 | -0.394 |
| 18 | -0.518 | 0.318 | -0.158 | -0.143 |
| 19 | -0.443 | 0.099 | -0.102 | -0.135 |
| 20 | -0.616 | -0.133 | -0.137 | -0.038 |
| 21 | -0.594 | 0.212 | -0.072 | 0.137 |
| 22 | -0.611 | -0.105 | -0.121 | -0.031 |
| 23 | -0.689 | -0.062 | -0.148 | 0.103 |
| 24 | -0.651 | 0.170 | 0.189 | 0.000 |

UNIQUE VARIANCES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.450 | 0.770 | 0.662 | 0.650 | 0.361 | 0.324 | 0.271 | 0.487 | 0.256 | 0.257 |
| 0.530 | 0.448 | 0.489 | 0.636 | 0.693 | 0.549 | 0.586 | 0.585 | 0.765 | 0.583 |
| 0.578 | 0.600 | 0.488 | 0.512 | | | | | | |

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.150 | 0.197 | 0.682 | 0.153 |
| 2 | 0.113 | 0.082 | 0.452 | 0.076 |
| 3 | 0.147 | -0.011 | 0.551 | 0.114 |
| 4 | 0.230 | 0.090 | 0.533 | 0.070 |
| 5 | 0.731 | 0.216 | 0.194 | 0.143 |
| 6 | 0.757 | 0.071 | 0.212 | 0.230 |
| 7 | 0.814 | 0.153 | 0.192 | 0.073 |
| 8 | 0.568 | 0.230 | 0.343 | 0.138 |
| 9 | 0.809 | 0.051 | 0.199 | 0.218 |
| 10 | 0.171 | 0.824 | -0.106 | 0.156 |
| 11 | 0.176 | 0.539 | 0.105 | 0.371 |
| 12 | 0.024 | 0.709 | 0.200 | 0.091 |
| 13 | 0.179 | 0.542 | 0.422 | 0.080 |
| 14 | 0.206 | 0.083 | 0.049 | 0.559 |
| 15 | 0.119 | 0.077 | 0.116 | 0.523 |
| 16 | 0.072 | 0.057 | 0.419 | 0.517 |
| 17 | 0.139 | 0.223 | 0.063 | 0.584 |
| 18 | 0.021 | 0.342 | 0.306 | 0.451 |
| 19 | 0.146 | 0.178 | 0.245 | 0.349 |
| 20 | 0.381 | 0.104 | 0.419 | 0.292 |
| 21 | 0.179 | 0.429 | 0.404 | 0.206 |
| 22 | 0.367 | 0.131 | 0.406 | 0.288 |
| 23 | 0.375 | 0.231 | 0.518 | 0.223 |
| 24 | 0.365 | 0.487 | 0.187 | 0.287 |

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.7646D 01 | - - |
| 2 | 0.1690D 01 | 0.5956D 01 |
| 3 | 0.1218D 01 | 0.4719D 00 |
| 4 | 0.9157D 00 | 0.3021D 00 |
| 5 | 0.4033D 00 | 0.5124D 00 |
| 6 | 0.3560D 00 | 0.4730D-01 |
| 7 | 0.2792D 00 | 0.7680D-01 |
| 8 | 0.2656D 00 | 0.1361D-01 |
| 9 | 0.2348D 00 | 0.3081D-01 |
| 10 | 0.1367D 00 | 0.9805D-01 |
| 11 | 0.7917D-01 | 0.5753D-01 |
| 12 | 0.4465D-01 | 0.3452D-01 |
| 13 | 0.1313D-01 | 0.3152D-01 |
| 14 | -0.2655D-01 | 0.3968D-01 |
| 15 | -0.3890D-01 | 0.1235D-01 |
| 16 | -0.7655D-01 | 0.3765D-01 |
| 17 | -0.1009D 00 | 0.2432D-01 |
| 18 | -0.1343D 00 | 0.3341D-01 |
| 19 | -0.1714D 00 | 0.3709D-01 |
| 20 | -0.1855D 00 | 0.1410D-01 |
| 21 | -0.2051D 00 | 0.1959D-01 |
| 22 | -0.2455D 00 | 0.4046D-01 |
| 23 | -0.3018D 00 | 0.5629D-01 |
| 24 | -0.3260D 00 | 0.2420D-01 |



F21P4000-I.N 34805

C 43508

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | -0.060 | 1.000 | | | | | | | | |
| 3 | -0.019 | 0.061 | 1.000 | | | | | | | |
| 4 | 0.077 | -0.070 | -0.045 | 1.000 | | | | | | |
| 5 | 0.035 | 0.163 | 0.038 | -0.152 | 1.000 | | | | | |
| 6 | 0.073 | 0.059 | 0.031 | 0.039 | -0.060 | 1.000 | | | | |
| 7 | 0.025 | -0.088 | -0.022 | 0.063 | -0.063 | 0.125 | 1.000 | | | |
| 8 | -0.116 | -0.150 | 0.169 | 0.084 | 0.063 | -0.061 | 0.123 | 1.000 | | |
| 9 | 0.074 | -0.016 | -0.168 | 0.032 | 0.168 | 0.020 | -0.134 | -0.107 | 1.000 | |
| 10 | -0.071 | 0.014 | -0.122 | 0.076 | 0.021 | 0.004 | -0.042 | 0.036 | -0.093 | 1.000 |
| 11 | 0.096 | 0.016 | -0.049 | -0.104 | 0.059 | 0.179 | -0.108 | -0.022 | 0.023 | -0.099 |
| 12 | 0.045 | -0.023 | 0.044 | -0.041 | -0.019 | -0.097 | 0.021 | 0.027 | -0.008 | 0.011 |
| 13 | 0.117 | -0.037 | 0.104 | 0.012 | 0.007 | 0.069 | 0.080 | 0.026 | 0.018 | -0.105 |
| 14 | -0.077 | 0.012 | 0.087 | -0.084 | 0.046 | -0.019 | 0.013 | 0.039 | -0.105 | -0.035 |
| 15 | 0.082 | 0.026 | -0.112 | -0.009 | 0.056 | 0.022 | 0.006 | -0.039 | 0.024 | 0.003 |
| 16 | 0.054 | 0.046 | -0.062 | 0.068 | -0.075 | 0.059 | -0.016 | 0.051 | -0.042 | 0.075 |
| 17 | -0.041 | -0.152 | 0.092 | 0.098 | -0.082 | 0.009 | 0.064 | 0.042 | 0.025 | -0.008 |
| 18 | 0.038 | 0.077 | -0.014 | 0.033 | 0.109 | -0.095 | -0.006 | -0.015 | 0.036 | -0.016 |
| 19 | -0.013 | -0.074 | 0.165 | -0.096 | -0.100 | -0.008 | 0.016 | 0.029 | 0.049 | -0.022 |
| 20 | -0.085 | 0.043 | -0.036 | -0.003 | -0.056 | -0.038 | 0.058 | 0.005 | -0.038 | 0.053 |
| 21 | -0.098 | 0.078 | -0.166 | 0.064 | -0.029 | -0.082 | 0.025 | -0.011 | -0.067 | 0.082 |
| 22 | 0.022 | -0.038 | -0.094 | 0.076 | 0.052 | -0.121 | -0.055 | -0.113 | 0.091 | -0.034 |
| 23 | -0.033 | 0.058 | 0.034 | -0.113 | -0.050 | -0.076 | -0.141 | 0.054 | 0.105 | 0.078 |
| 24 | -0.084 | 0.037 | 0.033 | 0.001 | -0.067 | 0.041 | 0.022 | -0.071 | 0.012 | 0.116 |

RESIDUAL CORRELATIONS

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 1.000 | | | | | | | | | |
| 12 | -0.027 | 1.000 | | | | | | | | |
| 13 | 0.269 | 0.067 | 1.000 | | | | | | | |
| 14 | 0.097 | 0.012 | 0.085 | 1.000 | | | | | | |
| 15 | -0.047 | 0.081 | -0.026 | 0.062 | 1.000 | | | | | |
| 16 | 0.064 | -0.109 | 0.036 | 0.141 | -0.012 | 1.000 | | | | |
| 17 | -0.011 | 0.099 | -0.047 | -0.059 | -0.003 | -0.047 | 1.000 | | | |
| 18 | -0.067 | 0.008 | -0.059 | -0.162 | 0.053 | -0.068 | 0.147 | 1.000 | | |
| 19 | 0.021 | -0.171 | 0.015 | -0.066 | -0.069 | -0.071 | 0.067 | 0.092 | 1.000 | |
| 20 | -0.132 | 0.103 | -0.157 | 0.051 | 0.027 | 0.049 | -0.019 | -0.005 | -0.167 | 1.000 |
| 21 | 0.031 | -0.105 | -0.051 | -0.040 | 0.036 | 0.062 | -0.160 | -0.018 | 0.087 | 0.122 |
| 22 | 0.034 | -0.031 | -0.096 | -0.039 | -0.009 | -0.121 | -0.002 | 0.018 | 0.096 | 0.094 |
| 23 | -0.150 | 0.114 | -0.095 | -0.008 | 0.029 | -0.025 | 0.038 | -0.139 | 0.004 | 0.112 |
| 24 | -0.078 | -0.008 | -0.147 | 0.033 | -0.147 | -0.036 | -0.024 | 0.080 | 0.141 | 0.026 |

RESIDUAL CORRELATIONS

| | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| 21 | 1.000 | | | |
| 22 | 0.049 | 1.000 | | |
| 23 | 0.056 | 0.112 | 1.000 | |
| 24 | 0.072 | 0.033 | 0.048 | 1.000 |



C-43306-B

UNWEIGHTED LEAST SQUARES SOLUTION FOR 5 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | -0.598 | -0.024 | -0.381 | 0.210 | 0.085 |
| 2 | -0.372 | 0.035 | -0.261 | 0.136 | -0.071 |
| 3 | -0.420 | 0.125 | -0.366 | 0.128 | 0.117 |
| 4 | -0.483 | 0.112 | -0.258 | 0.183 | -0.060 |
| 5 | -0.686 | 0.293 | 0.279 | 0.044 | 0.027 |
| 6 | -0.687 | 0.402 | 0.212 | -0.063 | 0.140 |
| 7 | -0.677 | 0.407 | 0.310 | 0.089 | 0.029 |
| 8 | -0.673 | 0.194 | 0.094 | 0.109 | 0.037 |
| 9 | -0.695 | 0.445 | 0.235 | -0.066 | -0.000 |
| 10 | -0.477 | -0.541 | 0.476 | 0.085 | -0.170 |
| 11 | -0.562 | -0.376 | 0.168 | -0.093 | 0.271 |
| 12 | -0.468 | -0.494 | 0.125 | 0.219 | -0.065 |
| 13 | -0.618 | -0.306 | -0.028 | 0.371 | 0.378 |
| 14 | -0.425 | -0.051 | -0.003 | -0.422 | 0.146 |
| 15 | -0.390 | -0.084 | -0.092 | -0.369 | -0.003 |
| 16 | -0.510 | -0.075 | -0.346 | -0.251 | 0.074 |
| 17 | -0.464 | -0.195 | 0.004 | -0.394 | 0.026 |
| 18 | -0.518 | -0.307 | -0.165 | -0.160 | -0.081 |
| 19 | -0.442 | -0.093 | -0.102 | -0.136 | 0.030 |
| 20 | -0.619 | 0.147 | -0.141 | -0.055 | -0.225 |
| 21 | -0.596 | -0.210 | -0.080 | 0.119 | -0.180 |
| 22 | -0.611 | 0.113 | -0.120 | -0.042 | -0.141 |
| 23 | -0.691 | 0.071 | -0.152 | 0.092 | -0.182 |
| 24 | -0.652 | -0.168 | 0.187 | -0.017 | -0.158 |

UNIQUE VARIANCES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.445 | 0.769 | 0.644 | 0.651 | 0.362 | 0.298 | 0.272 | 0.487 | 0.260 | 0.217 |
| 0.431 | 0.469 | 0.242 | 0.617 | 0.696 | 0.546 | 0.591 | 0.579 | 0.766 | 0.522 |
| 0.548 | 0.578 | 0.452 | 0.487 | | | | | | |

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0.155 | 0.147 | 0.666 | 0.168 | 0.192 |
| 2 | 0.111 | 0.079 | 0.455 | 0.075 | -0.006 |
| 3 | 0.151 | -0.059 | 0.537 | 0.126 | 0.159 |
| 4 | 0.228 | 0.082 | 0.535 | 0.068 | 0.013 |
| 5 | 0.731 | 0.198 | 0.197 | 0.148 | 0.052 |
| 6 | 0.769 | 0.034 | 0.199 | 0.241 | 0.112 |
| 7 | 0.815 | 0.138 | 0.195 | 0.075 | 0.044 |
| 8 | 0.571 | 0.203 | 0.342 | 0.146 | 0.093 |
| 9 | 0.805 | 0.050 | 0.208 | 0.213 | -0.028 |
| 10 | 0.173 | 0.849 | -0.088 | 0.157 | 0.002 |
| 11 | 0.188 | 0.477 | 0.071 | 0.419 | 0.353 |
| 12 | 0.030 | 0.678 | 0.208 | 0.106 | 0.124 |
| 13 | 0.186 | 0.476 | 0.415 | 0.086 | 0.563 |
| 14 | 0.210 | 0.053 | 0.042 | 0.573 | 0.081 |
| 15 | 0.118 | 0.078 | 0.127 | 0.516 | -0.044 |
| 16 | 0.074 | 0.031 | 0.414 | 0.521 | 0.066 |
| 17 | 0.141 | 0.213 | 0.073 | 0.582 | 0.003 |
| 18 | 0.019 | 0.340 | 0.320 | 0.450 | -0.015 |
| 19 | 0.147 | 0.161 | 0.246 | 0.351 | 0.051 |
| 20 | 0.373 | 0.135 | 0.455 | 0.275 | -0.195 |
| 21 | 0.173 | 0.445 | 0.428 | 0.198 | -0.048 |
| 22 | 0.362 | 0.147 | 0.426 | 0.277 | -0.106 |
| 23 | 0.369 | 0.246 | 0.546 | 0.209 | -0.098 |
| 24 | 0.363 | 0.503 | 0.209 | 0.284 | -0.063 |

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.7674D 01 | - - |
| 2 | 0.1717D 01 | 0.5957D 01 |
| 3 | 0.1233D 01 | 0.4837D 00 |
| 4 | 0.9487D 00 | 0.2847D 00 |
| 5 | 0.4981D 00 | 0.4506D 00 |
| 6 | 0.3678D 00 | 0.1302D 00 |
| 7 | 0.2958D 00 | 0.7207D-01 |
| 8 | 0.2727D 00 | 0.2309D-01 |
| 9 | 0.2407D 00 | 0.3195D-01 |
| 10 | 0.1577D 00 | 0.8302D-01 |
| 11 | 0.9655D-01 | 0.6118D-01 |
| 12 | 0.6441D-01 | 0.3214D-01 |
| 13 | 0.3177D-01 | 0.3264D-01 |
| 14 | 0.8511D-02 | 0.2326D-01 |
| 15 | -0.3545D-02 | 0.1206D-01 |
| 16 | -0.3796D-01 | 0.3441D-01 |
| 17 | -0.7134D-01 | 0.3339D-01 |
| 18 | -0.1037D 00 | 0.3239D-01 |
| 19 | -0.1436D 00 | 0.3989D-01 |
| 20 | -0.1644D 00 | 0.2078D-01 |
| 21 | -0.1871D 00 | 0.2271D-01 |
| 22 | -0.2263D 00 | 0.3923D-01 |
| 23 | -0.2838D 00 | 0.5744D-01 |
| 24 | -0.3141D 00 | 0.3035D-01 |

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | -0.043 | 1.000 | | | | | | | | |
| 3 | -0.040 | 0.074 | 1.000 | | | | | | | |
| 4 | 0.094 | -0.070 | -0.035 | 1.000 | | | | | | |
| 5 | 0.031 | 0.167 | 0.033 | -0.148 | 1.000 | | | | | |
| 6 | 0.044 | 0.080 | -0.003 | 0.026 | -0.084 | 1.000 | | | | |
| 7 | 0.018 | -0.083 | -0.031 | 0.066 | -0.060 | 0.103 | 1.000 | | | |
| 8 | -0.121 | -0.143 | 0.163 | 0.090 | 0.064 | -0.084 | 0.122 | 1.000 | | |
| 9 | 0.073 | -0.019 | -0.168 | 0.029 | 0.174 | 0.014 | -0.124 | -0.104 | 1.000 | |
| 10 | -0.012 | -0.002 | -0.064 | 0.069 | 0.035 | 0.080 | -0.029 | 0.065 | -0.113 | 1.000 |
| 11 | 0.053 | 0.052 | -0.107 | -0.081 | 0.045 | 0.106 | -0.139 | -0.047 | 0.034 | -0.046 |
| 12 | 0.064 | -0.023 | 0.055 | -0.041 | -0.011 | -0.086 | 0.025 | 0.038 | -0.022 | 0.017 |
| 13 | -0.027 | -0.025 | -0.006 | 0.026 | -0.033 | -0.062 | 0.062 | -0.040 | 0.071 | -0.027 |
| 14 | -0.105 | 0.020 | 0.065 | -0.077 | 0.039 | -0.061 | 0.007 | 0.030 | -0.099 | 0.010 |
| 15 | 0.082 | 0.021 | -0.111 | -0.013 | 0.056 | 0.030 | 0.009 | -0.038 | 0.028 | -0.006 |
| 16 | 0.044 | 0.053 | -0.07 | 0.076 | -0.080 | 0.044 | -0.019 | 0.047 | -0.038 | 0.113 |
| 17 | -0.047 | -0.154 | 0.090 | 0.095 | -0.082 | 0.006 | 0.064 | 0.040 | 0.029 | -0.005 |
| 18 | 0.057 | 0.070 | 0.003 | 0.028 | 0.115 | -0.071 | 0.002 | -0.006 | 0.034 | -0.049 |
| 19 | -0.016 | -0.072 | 0.163 | -0.093 | -0.100 | -0.014 | 0.016 | 0.029 | 0.051 | -0.014 |
| 20 | -0.051 | 0.018 | 0.001 | -0.028 | -0.051 | 0.021 | 0.073 | 0.018 | -0.054 | -0.027 |
| 21 | -0.066 | 0.065 | -0.138 | 0.053 | -0.017 | -0.031 | 0.040 | 0.005 | -0.074 | 0.012 |
| 22 | 0.050 | -0.052 | -0.070 | 0.066 | 0.061 | -0.088 | -0.046 | -0.104 | 0.087 | -0.094 |
| 23 | 0.001 | 0.040 | 0.070 | -0.136 | -0.042 | -0.025 | -0.135 | 0.070 | 0.099 | 0.015 |
| 24 | -0.049 | 0.024 | 0.071 | -0.011 | -0.060 | 0.089 | 0.034 | -0.058 | 0.002 | 0.046 |

RESIDUAL CORRELATIONS

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 1.000 | | | | | | | | | |
| 12 | -0.010 | 1.000 | | | | | | | | |
| 13 | 0.027 | 0.054 | 1.000 | | | | | | | |
| 14 | 0.026 | 0.016 | 0.046 | 1.000 | | | | | | |
| 15 | -0.053 | 0.072 | 0.019 | 0.067 | 1.000 | | | | | |
| 16 | 0.029 | -0.106 | -0.004 | 0.127 | -0.007 | 1.000 | | | | |
| 17 | -0.033 | 0.098 | -0.042 | -0.061 | 0.004 | -0.047 | 1.000 | | | |
| 18 | -0.043 | 0.010 | -0.015 | -0.152 | 0.050 | -0.060 | 0.150 | 1.000 | | |
| 19 | 0.005 | -0.164 | -0.005 | -0.071 | -0.066 | -0.071 | 0.069 | 0.097 | 1.000 | |
| 20 | -0.023 | 0.088 | 0.014 | 0.098 | 0.015 | 0.070 | -0.021 | -0.044 | -0.171 | 1.000 |
| 21 | 0.119 | -0.110 | 0.037 | -0.008 | 0.029 | 0.080 | -0.163 | -0.044 | 0.095 | 0.056 |
| 22 | 0.115 | -0.043 | 0.003 | -0.014 | -0.015 | -0.110 | -0.002 | -0.004 | 0.103 | 0.032 |
| 23 | -0.062 | 0.105 | 0.021 | 0.031 | 0.021 | -0.007 | 0.041 | -0.174 | 0.010 | 0.027 |
| 24 | -0.017 | -0.008 | -0.072 | 0.063 | -0.159 | -0.021 | -0.023 | 0.060 | 0.150 | -0.045 |

RESIDUAL CORRELATIONS

| | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| 21 | 1.000 | | | |
| 22 | 0.007 | 1.000 | | |
| 23 | -0.004 | 0.063 | 1.000 | |
| 24 | 0.025 | -0.009 | -0.007 | 1.000 |

HARMAN'S 13 PSYCHOLOGICAL TESTS ML

N= 145

P= 24

KL= 4

KU= 4

MAXIT= 30

LOGICAL VARIABLES= TF

INTEGER VARIABLES= 3300

EPS= 0.0005000

EPSE= 0.1000000

LOGICAL SCRATCH TAPE (DISK) NUMBER= 4



C 43506 8

MATRIX TO BE ANALYZED

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | 0.318 | 1.000 | | | | | | | | |
| 3 | 0.403 | 0.317 | 1.000 | | | | | | | |
| 4 | 0.468 | 0.230 | 0.305 | 1.000 | | | | | | |
| 5 | 0.321 | 0.285 | 0.247 | 0.227 | 1.000 | | | | | |
| 6 | 0.335 | 0.234 | 0.268 | 0.327 | 0.622 | 1.000 | | | | |
| 7 | 0.304 | 0.157 | 0.223 | 0.335 | 0.656 | 0.722 | 1.000 | | | |
| 8 | 0.332 | 0.157 | 0.382 | 0.391 | 0.578 | 0.527 | 0.619 | 1.000 | | |
| 9 | 0.326 | 0.195 | 0.184 | 0.325 | 0.723 | 0.714 | 0.685 | 0.532 | 1.000 | |
| 10 | 0.116 | 0.057 | -0.075 | 0.099 | 0.311 | 0.203 | 0.246 | 0.285 | 0.170 | 1.000 |
| 11 | 0.308 | 0.150 | 0.091 | 0.110 | 0.344 | 0.353 | 0.232 | 0.300 | 0.280 | 0.484 |
| 12 | 0.314 | 0.145 | 0.140 | 0.160 | 0.215 | 0.095 | 0.181 | 0.271 | 0.113 | 0.585 |
| 13 | 0.489 | 0.239 | 0.321 | 0.327 | 0.344 | 0.309 | 0.345 | 0.395 | 0.280 | 0.408 |

MATRIX TO BE ANALYZED

| | 11 | 12 | 13 |
|---|---|---|---|
| 11 | 1.000 | | |
| 12 | 0.428 | 1.000 | |
| 13 | 0.535 | 0.512 | 1.000 |



F21P4000-I.N. 346008

MAXIMUM LIKELIHOOD SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.309 | 0.395 | -0.236 | -0.482 |
| 2 | 0.151 | 0.262 | -0.097 | -0.282 |
| 3 | 0.092 | 0.359 | -0.128 | -0.502 |
| 4 | 0.111 | 0.443 | -0.179 | -0.325 |
| 5 | 0.346 | 0.716 | 0.044 | 0.130 |
| 6 | 0.355 | 0.734 | 0.211 | 0.037 |
| 7 | 0.234 | 0.806 | 0.048 | 0.129 |
| 8 | 0.302 | 0.646 | -0.114 | -0.026 |
| 9 | 0.282 | 0.773 | 0.198 | 0.093 |
| 10 | 0.485 | 0.141 | -0.507 | 0.425 |
| 11 | 1.000 | -0.002 | 0.001 | -0.000 |
| 12 | 0.429 | 0.124 | -0.647 | 0.071 |
| 13 | 0.536 | 0.300 | -0.399 | -0.216 |

UNIQUE VARIANCES

0.461 0.820 0.595 0.654 0.349 0.289 0.277 0.478 0.276 0.307
0.000 0.378 0.417

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.140 | 0.186 | 0.154 | 0.679 |
| 2 | 0.061 | 0.141 | 0.046 | 0.393 |
| 3 | -0.002 | 0.135 | -0.032 | 0.621 |
| 4 | -0.048 | 0.254 | 0.090 | 0.521 |
| 5 | 0.105 | 0.753 | 0.185 | 0.197 |
| 6 | 0.177 | 0.791 | 0.015 | 0.233 |
| 7 | -0.018 | 0.809 | 0.142 | 0.217 |
| 8 | 0.050 | 0.586 | 0.236 | 0.347 |
| 9 | 0.090 | 0.823 | 0.020 | 0.194 |
| 10 | 0.162 | 0.209 | 0.784 | -0.091 |
| 11 | 0.887 | 0.204 | 0.394 | 0.126 |
| 12 | 0.112 | 0.028 | 0.742 | 0.243 |
| 13 | 0.279 | 0.186 | 0.476 | 0.494 |

CHISQUARE WITH 32 DEGREES OF FREEDOM IS 47.4241

PROBABILITY LEVEL IS 0.039

TUCKER'S RELIABILITY COEFFICIENT= 0.950

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.1669D 01 | - - |
| 2 | 0.1554D 01 | 0.1152D 00 |
| 3 | 0.1181D 01 | 0.3730D 00 |
| 4 | 0.1125D 01 | 0.5573D-01 |
| 5 | 0.1008D 01 | 0.1171D 00 |
| 6 | 0.8990D 00 | 0.1088D 00 |
| 7 | 0.8633D 00 | 0.3576D-01 |
| 8 | 0.7399D 00 | 0.1234D 00 |
| 9 | 0.7109D 00 | 0.2898D-01 |
| 10 | 0.3284D 00 | 0.3825D 00 |
| 11 | 0.2582D 00 | 0.7023D-01 |
| 12 | 0.9104D-01 | 0.1672D 00 |
| 13 | 0.4991D-03 | 0.9055D-01 |



F31P4000.I.N 348005
C I 43568

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | 0.015 | 1.000 | | | | | | | | |
| 3 | -0.074 | 0.079 | 1.000 | | | | | | | |
| 4 | 0.109 | -0.016 | -0.080 | 1.000 | | | | | | |
| 5 | 0.010 | 0.161 | 0.064 | -0.164 | 1.000 | | | | | |
| 6 | 0.007 | 0.039 | 0.042 | 0.028 | -0.128 | 1.000 | | | | |
| 7 | -0.037 | -0.101 | -0.041 | 0.007 | -0.066 | 0.115 | 1.000 | | | |
| 8 | -0.119 | -0.121 | 0.178 | 0.076 | 0.048 | -0.079 | 0.102 | 1.000 | | |
| 9 | 0.071 | -0.009 | -0.116 | 0.041 | 0.166 | 0.005 | -0.090 | -0.075 | 1.000 | |
| 10 | -0.012 | 0.035 | -0.051 | 0.068 | 0.029 | 0.063 | -0.040 | 0.003 | -0.051 | 1.000 |
| 11 | -0.000 | -0.001 | -0.000 | -0.000 | -0.000 | 0.001 | -0.001 | 0.002 | -0.001 | -0.000 |
| 12 | 0.033 | 0.009 | 0.019 | -0.071 | -0.007 | -0.043 | 0.008 | -0.024 | 0.055 | 0.005 |
| 13 | 0.015 | -0.034 | 0.010 | -0.013 | -0.027 | -0.026 | 0.074 | -0.025 | -0.010 | -0.012 |

RESIDUAL CORRELATIONS

| | 11 | 12 | 13 |
|---|---|---|---|
| 11 | 1.000 | | |
| 12 | -0.000 | 1.000 | |
| 13 | 0.001 | 0.006 | 1.000 |

HARMAN'S 12 PSYCHOLOGICAL TESTS ML

N= 145

P= 24

KL= 3

KU= 4

MAXIT= 30

LOGICAL VARIABLES= TF

INTEGER VARIABLES= 3300

EPS= 0.0005000

EPSE= 0.1000000

LOGICAL SCRATCH TAPE (DISK) NUMBER= 4



F2I P4000-1 N 348003

MATRIX TO BE ANALYZED

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | 0.408 | 1.000 | | | | | | | | |
| 3 | 0.512 | 0.585 | 1.000 | | | | | | | |
| 4 | 0.344 | 0.311 | 0.215 | 1.000 | | | | | | |
| 5 | 0.327 | 0.099 | 0.160 | 0.227 | 1.000 | | | | | |
| 6 | 0.280 | 0.170 | 0.113 | 0.723 | 0.325 | 1.000 | | | | |
| 7 | 0.489 | 0.116 | 0.314 | 0.321 | 0.468 | 0.326 | 1.000 | | | |
| 8 | 0.395 | 0.285 | 0.271 | 0.578 | 0.391 | 0.532 | 0.332 | 1.000 | | |
| 9 | 0.345 | 0.246 | 0.181 | 0.656 | 0.335 | 0.685 | 0.304 | 0.619 | 1.000 | |
| 10 | 0.239 | 0.057 | 0.145 | 0.285 | 0.230 | 0.195 | 0.318 | 0.157 | 0.157 | 1.000 |
| 11 | 0.309 | 0.203 | 0.095 | 0.622 | 0.327 | 0.714 | 0.335 | 0.527 | 0.722 | 0.234 |
| 12 | 0.321 | -0.075 | 0.140 | 0.247 | 0.305 | 0.184 | 0.403 | 0.382 | 0.223 | 0.317 |

MATRIX TO BE ANALYZED

| | 11 | 12 |
|---|---|---|
| 11 | 1.000 | |
| 12 | 0.268 | 1.000 |



FB1P4000-I.N 848008

MAXIMUM LIKELIHOOD SOLUTION FOR 3 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0.550 | 0.416 | -0.278 |
| 2 | 0.418 | 0.659 | 0.333 |
| 3 | 0.378 | 0.680 | -0.057 |
| 4 | 0.793 | -0.110 | 0.134 |
| 5 | 0.449 | 0.005 | -0.354 |
| 6 | 0.795 | -0.289 | 0.128 |
| 7 | 0.506 | 0.124 | -0.517 |
| 8 | 0.714 | -0.003 | -0.054 |
| 9 | 0.807 | -0.184 | 0.122 |
| 10 | 0.307 | 0.021 | -0.294 |
| 11 | 0.786 | -0.258 | 0.074 |
| 12 | 0.357 | -0.049 | -0.521 |

UNIQUE VARIANCES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.448 | 0.281 | 0.391 | 0.342 | 0.673 | 0.267 | 0.461 | 0.487 | 0.300 | 0.819 |
| 0.310 | 0.598 | | | | | | | | |

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0.199 | 0.510 | 0.503 |
| 2 | 0.212 | 0.816 | -0.094 |
| 3 | 0.029 | 0.740 | 0.246 |
| 4 | 0.761 | 0.198 | 0.200 |
| 5 | 0.249 | 0.083 | 0.508 |
| 6 | 0.832 | 0.033 | 0.197 |
| 7 | 0.192 | 0.177 | 0.686 |
| 8 | 0.585 | 0.230 | 0.345 |
| 9 | 0.798 | 0.132 | 0.213 |
| 10 | 0.145 | 0.062 | 0.395 |
| 11 | 0.792 | 0.047 | 0.244 |
| 12 | 0.133 | -0.033 | 0.619 |

CHISQUARE WITH 33 DEGREES OF FREEDOM IS 46.5319

PROBABILITY LEVEL IS 0.059

TUCKER'S RELIABILITY COEFFICIENT= 0.961

F81P4000-I.N. 348008

C81/E

EBI-72

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.1667D 01 | - - |
| 2 | 0.1524D 01 | 0.1435D 00 |
| 3 | 0.1173D 01 | 0.3506D 00 |
| 4 | 0.1120D 01 | 0.5326D-01 |
| 5 | 0.1024D 01 | 0.9556D-01 |
| 6 | 0.9038D 00 | 0.1203D 00 |
| 7 | 0.8644D 00 | 0.3942D-01 |
| 8 | 0.7422D 00 | 0.1222D 00 |
| 9 | 0.7087D 00 | 0.3355D-01 |
| 10 | 0.3237D 00 | 0.3849D 00 |
| 11 | 0.2074D 00 | 0.1163D 00 |
| 12 | 0.7544D-01 | 0.1320D 00 |

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | -0.007 | 1.000 | | | | | | | | |
| 3 | 0.013 | -0.005 | 1.000 | | | | | | | |
| 4 | -0.022 | 0.022 | -0.007 | 1.000 | | | | | | |
| 5 | -0.037 | 0.060 | -0.065 | -0.168 | 1.000 | | | | | |
| 6 | -0.004 | -0.055 | 0.050 | 0.145 | 0.035 | 1.000 | | | | |
| 7 | 0.034 | -0.014 | 0.021 | 0.007 | 0.103 | 0.071 | 1.000 | | | |
| 8 | -0.025 | 0.016 | -0.001 | 0.046 | 0.090 | -0.083 | -0.121 | 1.000 | | |
| 9 | 0.032 | -0.039 | 0.022 | -0.062 | 0.038 | -0.090 | -0.050 | 0.127 | 1.000 | |
| 10 | -0.033 | 0.027 | -0.004 | 0.158 | -0.016 | -0.012 | 0.013 | -0.124 | -0.103 | 1.000 |
| 11 | 0.013 | 0.066 | -0.065 | -0.120 | 0.004 | 0.017 | 0.018 | -0.080 | 0.103 | 0.039 |
| 12 | 0.000 | -0.045 | 0.018 | 0.063 | -0.062 | -0.120 | -0.079 | 0.182 | -0.026 | 0.079 |

RESIDUAL CORRELATIONS

| | 11 | 12 |
|---|---|---|
| 11 | 1.000 | |
| 12 | 0.029 | 1.000 |

FEB 11

LB33-



FE1P4000-I.N 348005

MAXIMUM LIKELIHOOD SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.345 | -0.614 | 0.132 | 0.193 |
| 2 | 0.312 | -0.477 | 0.528 | -0.313 |
| 3 | 0.216 | -0.596 | 0.465 | 0.037 |
| 4 | 1.000 | 0.002 | 0.001 | 0.001 |
| 5 | 0.228 | -0.443 | -0.273 | 0.201 |
| 6 | 0.724 | -0.180 | -0.346 | -0.170 |
| 7 | 0.322 | -0.488 | -0.151 | 0.418 |
| 8 | 0.579 | -0.383 | -0.177 | -0.028 |
| 9 | 0.657 | -0.335 | -0.349 | -0.259 |
| 10 | 0.285 | -0.166 | -0.058 | 0.322 |
| 11 | 0.623 | -0.312 | -0.431 | -0.216 |
| 12 | 0.248 | -0.285 | -0.239 | 0.455 |

UNIQUE VARIANCES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.449 | 0.299 | 0.381 | 0.000 | 0.636 | 0.295 | 0.460 | 0.486 | 0.267 | 0.784 |
| 0.281 | 0.594 | | | | | | | | |

VARIMAX-ROTATED FACTOR LOADINGS

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.203 | 0.006 | 0.520 | 0.490 |
| 2 | 0.193 | 0.065 | 0.806 | -0.099 |
| 3 | 0.016 | 0.022 | 0.750 | 0.237 |
| 4 | 0.705 | 0.653 | 0.189 | 0.202 |
| 5 | 0.286 | -0.144 | 0.098 | 0.501 |
| 6 | 0.797 | 0.174 | 0.045 | 0.196 |
| 7 | 0.198 | 0.012 | 0.187 | 0.682 |
| 8 | 0.582 | 0.085 | 0.240 | 0.332 |
| 9 | 0.821 | 0.016 | 0.150 | 0.189 |
| 10 | 0.111 | 0.173 | 0.056 | 0.413 |
| 11 | 0.815 | -0.015 | 0.061 | 0.227 |
| 12 | 0.134 | 0.050 | -0.029 | 0.620 |

CHISQUARE WITH 24 DEGREES OF FREEDOM IS 28.9430

PROBABILITY LEVEL IS 0.222

TUCKER'S RELIABILITY COEFFICIENT= 0.980

| | LATENT ROOTS | FIRST DIFFERENCES |
|---|---|---|
| 1 | 0.1496D 01 | - - |
| 2 | 0.1361D 01 | 0.1350D 00 |
| 3 | 0.1134D 01 | 0.2268D 00 |
| 4 | 0.1049D 01 | 0.8531D-01 |
| 5 | 0.9802D 00 | 0.6840D-01 |
| 6 | 0.8757D 00 | 0.1045D 00 |
| 7 | 0.8269D 00 | 0.4879D-01 |
| 8 | 0.7197D 00 | 0.1073D 00 |
| 9 | 0.3503D 00 | 0.3694D 00 |
| 10 | 0.2276D 00 | 0.1227D 00 |
| 11 | 0.1748D 00 | 0.5281D-01 |
| 12 | 0.4983D-03 | 0.1743D 00 |

6351.79



F31F4000-I.N 348008

RESIDUAL CORRELATIONS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | | | |
| 2 | -0.006 | 1.000 | | | | | | | | |
| 3 | 0.007 | -0.001 | 1.000 | | | | | | | |
| 4 | -0.000 | 0.001 | -0.001 | 1.000 | | | | | | |
| 5 | -0.050 | 0.054 | -0.068 | -0.001 | 1.000 | | | | | |
| 6 | -0.006 | -0.042 | 0.048 | 0.001 | 0.045 | 1.000 | | | | |
| 7 | 0.037 | -0.019 | 0.020 | 0.001 | 0.097 | 0.064 | 1.000 | | | |
| 8 | -0.024 | 0.016 | 0.003 | 0.001 | 0.084 | -0.059 | -0.119 | 1.000 | | |
| 9 | 0.023 | -0.056 | 0.034 | 0.001 | -0.016 | -0.057 | -0.045 | 0.114 | 1.000 | |
| 10 | -0.027 | 0.041 | -0.001 | 0.001 | 0.015 | -0.014 | 0.002 | -0.118 | -0.051 | 1.000 |
| 11 | 0.002 | 0.069 | -0.052 | -0.002 | -0.066 | 0.071 | 0.020 | -0.097 | 0.004 | 0.104 |
| 12 | 0.009 | -0.047 | 0.023 | 0.000 | -0.056 | -0.124 | -0.081 | 0.187 | -0.002 | 0.057 |

RESIDUAL CORRELATIONS

| | 11 | 12 |
|---|---|---|
| 11 | 1.000 | |
| 12 | 0.049 | 1.000 |

## Appendix C

If the user wishes to change the number of variables, $p$, and/or the number of variables before selection, $p_0$, the MAIN program and subroutines REX, SELECT, NWTRAP, INCPSI and FCTGR need to be modified.

In the MAIN program the DIMENSIØN card should read as follows:

DIMENSIØN FMT(10),S(n),A(n),HEAD(20),YY(p),E(m),Y(p)

where $n = (p_0(p_0 + 1))/2$ and $m = (p(p + 1))/2$ .

In subroutine REX the DIMENSIØN card should be:

DIMENSIØN S(1),E(1),Y($p_0$),X($p_0$),FMT(10) .

In subroutine SELECT the DIMENSIØN card should be:

DIMENSIØN S(1),E(1),MM($p_0$) .

In subroutines NWTRAP, INCPSI and FCTGR the CØMMØN block KERN should read:

CØMMØN/KERN/G(p),V(p),VB(p),D2(p),S1(p),S2(p),S3(p),EPSU,
BND,IM(p),MØR,KP,MØRE,MAXTRY,P2,KP1 .

Caution: The following relationship between $p$ and $p_0$ must hold

$$p^2 \leq (p_0(p_0 + 1))/2 \quad .$$

END

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